Journal of Theoretical and Applied Vibration and Acoustics 6(2) 201-216 (2020)

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Journal of Theoretical and Applied

Vibration and Acoustics

journal homepage: http://tava.isav.ir

Stabilization of a 5-D hyperchaotic Rikitake system with unknown parameters

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ARTICLE INFO

Article history:
Received 25 May 2020

Received in revised form
5 June 2020

Accepted 10 July 2020

Available online 15 July 2020

Keywords:
Instability,

Nonlinear systems,

Chaos,

Synchronization,

Time delay.

ABSTRACT

In this paper, a nonlinear 5-D hyperchaotic Rikitake dynamic system has been taken into consideration. The hyperchaotic behavior of the model was proved, and the response of the system has been shown. Besides, in the case of existing parametric uncertainties in the system, it shows even more complex behavior. An adaptive control strategy to have stable behavior is synchronized for an uncertain hyperchaotic system with an identical 5-D system. The stability of the control law has been identified by using the Lyapunov stability theory. The numerical simulations are presented for the hyperchaotic Rikitake system with unknown parameters and a system with time-varying parameters to indicate the effectiveness of the proposed algorithm for a class of complex systems. Moreover, since there are often lags between the signals gained by the system and the signals that the controller receives, the control input with the time delay parameter is implemented in the model. Also, the results show the gradual transformation from an unstable system into a stable one.

1. Introduction

Chaos theory has been a matter of debate since its presentation by Pecora and Carrol in 1990 [1]. It describes the unstable periodic behavior in restorative nonlinear dynamical systems. Also, its technological applications have been foundin aerospace technology, the medical cyber-physical system, secure communication, chemical, biological systems, and many other scientific disciplines [2-5]. The strange attractor, excellent sensitivity to initial conditions, highly complex dynamics, self-similarity, and inner randomness are the common features of chaotic systems [6, 7]. The well-known 3-D weather prediction system that represented chaotic behavior was

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http://dx.doi.org/10.22064/tava.2021.127572.1166

proposed by Lorenz when he had been working on the atmospheric convection [8]; after that, a simpler 3-D chaotic system was constructed by Rössler [9]. So far, many other paradigms of those systems have been revealed like Chen system [10], Chen Lee- system [11], Cai system [12], Sundarapandian systems [13], Arneodo system [14], Sprott systems [15], Pham system [16], Vaidyanathan systems [17, 18], etc.

The synchronization of chaotic systems has attracted huge attention as a vital issue in treating dynamical systems with nonlinearity. According to the coupling configuration, the synchronization of chaotic systems is divided into master-slave synchronization, mutual synchronization, and control laws. These are used so that the output of the chaotic system, which is called the response or slave system, could track asymptotically with time another chaotic system which is called the output of the master or drive system. In other words, after the transient, both derive-response systems show similar behavior.

As a result of the complex behavior of chaotic systems, designing an effective controller that establishes synchronization has always been challenging. A wide variety of techniques have been developed in order to achieve chaos synchronization, including the backstepping design method [19], adaptive control [20-22], sliding mode control method [23, 24], passive control method [25], active control strategy [26], linear matrix inequality approach [27], nonlinear control technique [28], and many other approaches. For instance, in [29], a fuzzy robust controller was proposed to synchronize different uncertain and unexpected chaotic systems. . In [30], impulsive control for chaotic systems was designed to obtain finite-time and fixed-time synchronization. In [31], the feedback control for time-delayed chaotic systems was proposed in order to ensure the exponential synchronization of the system. In [32], the two adaptive robust control is implemented in order to synchronize uncertain chaotic systems with hysteresis quantizer input. In [33], the adaptive robust method was designed in order to achieve the full-order synchronization and the reduced order synchronization between two nonlinear chaotic systems without canceling out the nonlinearity of the system.

In this letter, we announce an adaptive synchronization and identification method to achieve synchronization for a fifteen-term 5-D hyperchaotic Rikitake dynamical system with an identical 5-D system that has five unknown parameters to considerably transform chaotic behavior to periodic behavior. The system can be obtained by adding two state feedback controls to the famed three-dimensional Rikitake dynamical system. This hyperchaotic model has been analysed by several scientists before, but in fact, to the best of our knowledge, it is for the first time that this method has been utilized. At first, the phase portrait of the 5-D hyper-chaotic system is depicted. By the presented method, synchronization can be guaranteed even if a time-delay parameter exists in the input of the model. Furthermore, the unstable behavior can be transformed into a stable one; by this strategy, Lyapunov stability theory is proposed to depict an adaptive controller.

The paper is formed as follows. First, the three-dimensional phase trajectory and the time series of the model are plotted and the equation of the 5-D hyperchaotic system is defined. Then a set of new formulations is proposed to identifying the unknown parameters and synchronizing them with the desired system. Finally, numerical simulations are applied to illustrate the effectiveness of the suggested approach.

2. Mathematical Model of a 5-D hyperchaotic Rikitake system

The unified five-dimensional hyperchaotic system derived by adding two-state feedback controls to the 3-D Rikitake model given by [34],

$$\begin{cases} \dot{x}_1 = -\theta_1 x_1 + x_2 x_3 - \theta_4 x_4 + \theta_5 x_5 \\ \dot{x}_2 = -\theta_1 x_2 + x_1 (x_3 - \theta_2) - \theta_4 x_4 + \theta_5 x_5 \\ \dot{x}_3 = 1 - x_1 x_2 \\ \dot{x}_4 = \theta_3 x_2 \\ \dot{x}_5 = \theta_5 (x_1 + x_2 + x_4) \end{cases} \tag{1}$$

Where x_i (i = 1, … , 5) are the state variables and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ are positive constant parameters.

For studying uncontrolled system, the parameter values are taken as, θ_1= 1, θ_2= 1, θ_3=0.7, θ_4= 1.1, θ_5 = 0.1.

Fig. **1** shows the 3-D phase projection of the 5-D hyperchaotic Rikitake system (1) in (x_1, x_2, x_3) space. Fig. **2** indicates the time evolutions of the hyperchaotic model. The initial states of the hyperchaotic system (1) are taken as $x_1(0) = 2.4$, $x_2(0) = -1.2$, $x_3(0) = 0.6$, $x_4(0) = 1.9$, $x_5(0) = -2.3$. The system has no equilibrium points, while represent hidden attractors.

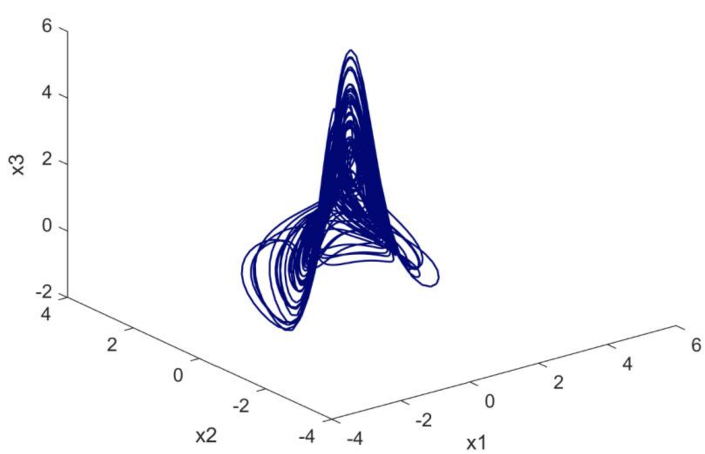

Fig. 1: The 3-D projection of the 5-D hyperchaotic system on (x1,x2,x3) space [34].

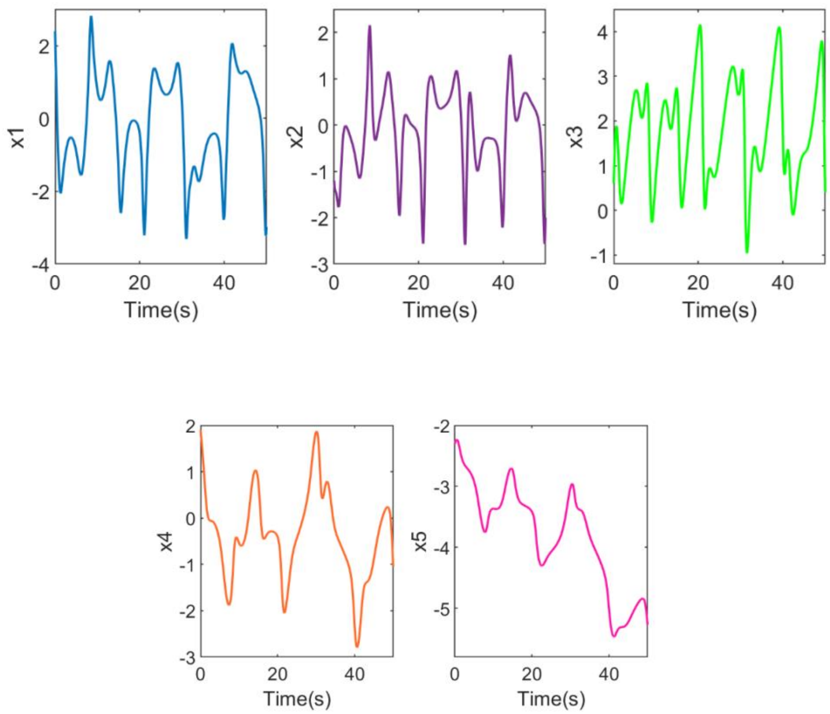

Fig. 2: Time evolution of the 5-D hyperchaotic system [34].

3. Adaptive synchronization of the uncertain 5-D hyperchaotic Rikitake system

Given two equal hyperchaotic systems, one is called a master system and the other is the slave system. Master-slave synchronization is used to couple two systems so that the behaviors of one system are controlled by the other one. In other words, the controlled (slave) system got a drive signal from the master system [35]. In this section, the general formulation for adaptive synchronization will be discussed. Then, the implementation of adaptive synchronization for the uncertain 5-D hyperchaotic Rikitake and an unknown time-varying system will be defined. Finally, a time-delayed control input for these systems will be presented.

3.1.The problem formulation

In this paper, the hyperchaotic driver system is assumed to be in the form of

$$\dot{x} = f(x) + F(x)\theta, \tag{1}$$

Where $f \in C^1(R^n, R^n)$ and $F \in C^1(R^n, R^{n\times m})$ are functions, $x \in R^n$ is the state vector of the system, and $\theta \in R^n$ is the known parameter. The controlled (response) system is given by

$$\dot{u} = f(u) + F(u)\alpha. \tag{3}$$

The structure of the equations for the slave system is similar to the master hyperchaotic system (2) structure, but $\alpha \in R^m$ is a parameter vector with full uncertainty.

In practice, α is unknown for the controlled system, but the output signal of the master system (2) could be transmitted to the slave system (3), So we wish to design the best-controller U for the slave system, such that the slave system.

$$\dot{u} = f(u) + F(u)\alpha + U, \tag{4}$$

where synchronous with the master system (2) and the uncertain parameter in the response system (3) is identified concurrently.

3.2.Synchronization of unknown 5-D hyperchaotic systems

We define Eq. (1) as a master system and Eq. (5) as a slave system as follows:

$$\begin{aligned}
\dot{y}_1 &= -\alpha_1 y_1 + y_2 y_3 - \alpha_4 y_4 + \alpha_5 y_5 + u_1 \\
\dot{y}_2 &= -\alpha_1 y_2 + y_1(y_3 - \alpha_2) - \alpha_4 y_4 + \alpha_5 y_5 + u_2 \\
\dot{y}_3 &= 1 - y_1 y_2 + u_3 \\
\dot{y}_4 &= \alpha_3 y_2 + u_4 \\
\dot{y}_5 &= \alpha_5(y_1 + y_2 + y_4) + u_5
\end{aligned} \tag{5}$$

where y_1, y_2, y_3, y_4, y_5 are state variables, but α_1, $\alpha_2, \alpha_3, \alpha_4, \alpha_5$ are unknown parameters, and u_1, u_2, u_3, u_4, u_5 are adaptive controls.

We propose an adaptive controller for the hyperchaotic Rikitake dynamo system (1), where all the system parameters are unknown and need to be identified.

The synchronization error between the hyperchaotic system (1) and response system (5) is defined as:

$$e_i = y_i - x_i \ (i = 1 \dots 5) \tag{6}$$

Then, if the system parameter θ is available, The following Lyapunov function is proposed to achieve an adaptive controller for the system.

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2). \tag{7}$$

Differentiating V function and replacing Eq. (6) and Selecting

$$
\begin{gathered}
u_1=(\theta_1-1)e_1+\theta_4 e_4-2\theta_5 e_5-x_3 e_2,\\
u_2=(\theta_1-1)e_2+(\theta_2-y_3)e_1+\theta_4 e_4-2\theta_5 e_5,\\
u_3=-e_3,\\
u_4=-e_4-\theta_3 e_2,\\
u_5=-e_5-\theta_5 e_4.
\end{gathered} \tag{8}
$$

We have

$$
\frac{dV}{dt}=-e_1^2-e_2^2-e_3^2-e_4^2-e_5^2\leq -2V(e). \tag{2}
$$

It is seen that $\dot{V}$ is a negative semi-definite function.

This adaptive controller and parameters identification update law are used to accomplish parameter estimation and synchronization when θ_i $(i=1\dots5)$ are not known. In this paper, we will use α instead of θ to show uncertainty. An updated law is proposed as follows:

$$
\dot{\alpha}(t)=-F^T(x)\big(grandV(e)\big)^T, \tag{10}
$$

In which

$$
\begin{gathered}
\dot{\alpha}_1=x_1 e_1+x_2 e_2,\\
\dot{\alpha}_2=x_1 e_2,\\
\dot{\alpha}_3=-x_2 e_4,\\
\dot{\alpha}_4=x_4 e_1+x_4 e_2,\\
\dot{\alpha}_5=-(x_1+x_2+x_4)e_5-x_5 e_1-x_5 e_2.
\end{gathered} \tag{11}
$$

Where

$$
\begin{gathered}
u_1=(\alpha_1-1)e_1+\alpha_4 e_4-2\alpha_5 e_5-x_3 e_2,\\
u_2=(\alpha_1-1)e_2+(\alpha_2-y_3)e_1+\alpha_4 e_4-2\alpha_5 e_5,\\
u_3=-e_3,\\
u_4=-e_4-\alpha_3 e_2,\\
u_4=-e_4-\alpha_3 e_2,
\end{gathered} \tag{12}
$$

These adaptive controls are applied for synchronization and tracking. The equation of the error system according to the master system (2) and the slave system (3) is:

$$\dot{e} = f(u) - f(x) + F(u)\alpha - F(x)\theta + U. \tag{13}$$

Then, we choose a Lyapunov function for this system.

$$V_1(e,\alpha) = V(e) + \frac{1}{2}(\alpha-\theta)^T(\alpha-\theta), \tag{14}$$

Where $V(e)$ is a presumption function. From Eq. (14) and the parameter identification update law (10) and replacing the solution of slave system (3), the time derivative of V_1 function will be

$$\begin{aligned}\frac{dV_1}{dt} &= (grandV(e), f(u) - f(x) + F(u)\alpha - F(x)\theta + U) + \dot{\alpha}^T(\alpha-\theta)\\ &= (grandV(e), f(u) - f(x) + F(u)\alpha - F(x)\theta + U)\\ &+ \big(grandV(e), F(x)(\alpha-\theta)\big) + \dot{\alpha}^T(\alpha-\theta) \le -2V(e) = W(e).\end{aligned} \tag{15}$$

Since $V_1(e,\alpha)$ is a positive definite function and $\frac{dV_1}{dt}$ is a negative semi-definite one, the stability of the points $\alpha = 0$ and $e = 0$ are demonstrated. This concludes that identification update laws (10) and the error system (13) are asymptotically stable accordingly, $e(t)$ and $\alpha(t)$ are bounded on the interval $(0,+\infty)$. Let $\overline{lim}_{t\to\infty} e(t) = \bar{e}$, $\overline{lim}_{t\to\infty}\alpha(t) = \bar{\alpha}$. If $e \neq 0$, then there exist two positive constants δ, ε, such that $\|e(t) - \bar{e}\| < \delta$ implies $W(e) > \varepsilon$. By the explanation of superior limit, there exists a sequence $\{t_n\} \subset R^+$ such that $\big(e(t_n),\ \alpha(t_n)\big) \to (\bar{e},\ \bar{\alpha})\ as\ n \to\ \infty$. Presume n^* to be the integer such that $\|e(t) - \bar{e}\| < \frac{1}{2}\delta$ for any $n > n^*$.Then, by the continuity of $V_1(e,\alpha)$, for n large enough, we have

$$V_1(e(t_n),\alpha(t_n)) - V_1(\bar{e},\bar{\alpha}) < \frac{\varepsilon\delta}{4r},$$

where $\frac{\varepsilon\delta}{4r}$ is a fixed picked outnumber, such that on $(t_n, t_n + \frac{\delta}{3r})$ we have $\|e(t) - \bar{e}\| < \delta$ and also $W(e) > \varepsilon$.

Consequently,

$$V_1(e(t_n),\alpha(t_n)) - V_1(\bar{e},\bar{\alpha}) \ge \int_{t_n}^{t_n+\frac{\delta}{3r}} W(e)\,dt > \frac{\varepsilon\delta}{3r}.$$

Then, we get inconsistency, which implies $\overline{lim}_{t\to\infty} e(t) = 0$. Likewise, we have $\underline{lim}_{t\to\infty} e(t) = 0$. Therefore, $lim_{t\to\infty} e(t) = 0$.

Furthermore, we can easily understand from the error of the dynamical system (13) that $\ddot{e}(t)$ is bounded, so that is proof of the claim that $lim_{t\to\infty}\dot{e}(t) = 0$. Accordingly, the slave system (5) is synchronized with the master system (1) and satisfies the convergence of the system's parameters $lim_{t\to\infty}\|\alpha(t) - \theta\| = 0$.

3.3.Synchronization of unknown 5-d non-autonomous systems

In this step, the derived adaptive controls are used for another master system which has time-varying parameters.

The new time-variant system is

$$\begin{aligned}
&\dot{x}_1 = -\theta_1 x_1 + x_2 x_3 - \theta_4 x_4 + \theta_5 x_5 \\
&\dot{x}_2 = -\theta_1 x_2 + x_1\big(x_3 - (1 - e^{-\lambda t})\theta_2\big) - \theta_4 x_4 + \theta_5 x_5 \\
&\dot{x}_3 = 1 - x_1 x_2 \\
&\dot{x}_4 = \left(1 - e^{-\lambda t}\right)\theta_3 x_2 \\
&\dot{x}_5 = \theta_5(x_1 + x_2 + x_4),
\end{aligned} \tag{16}$$

where x1, x2, x3, x4, x5 are the state variables, $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ are positive constant parameters and λ is a positive constant parameter. The process like what is defined in the previous section. Eq. (16) is assumed as a master system and Eq. (17) as a slave system:

$$\begin{cases}
\dot{y}_1 = -\alpha_1 y_1 + y_2 y_3 - \alpha_4 y_4 + \alpha_5 y_5 + u_1 \\
\dot{y}_2 = -\alpha_1 y_2 + y_1\big(y_3 - \left(1 - e^{-\lambda t}\right)\alpha_2\big) - \alpha_4 y_4 + \alpha_5 y_5 + u_2 \\
\dot{y}_3 = 1 - y_1 y_2 + u_3 \\
\dot{y}_4 = \left(1 - e^{-\lambda t}\right)\alpha_3 y_2 + u_4 \\
\dot{y}_5 = \alpha_5(y_1 + y_2 + y_4) + u_5
\end{cases} \tag{17}$$

where y_1, y_2, y_3, y_4, y_5 are state variables, but $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are unknown parameters, and u_1, u_2, u_3, u_4, u_5 are adaptive controls.

Through Lyapunov function that is discussed in the pervious section and parameters identification update law, an adaptive control input can be proposed as follows:

$$\begin{gathered}
\dot{\alpha}_1 = x_1 e_1 + x_2 e_2, \\
\dot{\alpha}_2 = x_1 e_2, \\
\dot{\alpha}_3 = -x_2 e_4, \\
\dot{\alpha}_4 = x_4 e_1 + x_4 e_2, \\
\dot{\alpha}_5 = -(x_1 + x_2 + x_4)e_5 - x_5 e_1 - x_5 e_2. \\
u_1 = (\alpha_1 - 1)e_1 + \alpha_4 e_4 - 2\alpha_5 e_5 - x_3 e_2, \\
u_2 = (\alpha_1 - 1)e_2 + \left(\left(1 - e^{-\lambda t}\right)\alpha_2 - y_3\right)e_1 + \alpha_4 e_4 - 2\alpha_5 e_5,
\end{gathered} \tag{18}$$

$$\begin{gathered}
u_3 = -e_3, \\
u_4 = -e_4 - \left(1 - e^{-\lambda t}\right)\alpha_3 e_2, \\
u_5 = -e_5 - \alpha_5 e_4.
\end{gathered} \tag{19}$$

3.4.Time-delayed adaptive synchronization

The structure of the control systems is based on the feedback of using from the state variables of the dynamic model. Moreover, the reality is that controllers have been unable to receive this feedback from the system at the same time. Hence, the control systems have a lack of receiving all the information from the model. The control input with delay for the unknown 5-d hyperchaotic system is presented as follows:

$$\begin{gathered}
u_1 = (\alpha_1 - 1)(y_1(t-\tau) - x_1) + \alpha_4(y_4(t-\tau) - x_4) - 2\alpha_5(y_5(t-\tau) - x_5) - \\
x_3(y_2(t-\tau) - x_2), \\
u_2 = (\alpha_1 - 1)(y_2(t-\tau) - x_2) + (\alpha_2 - y_3)(y_1(t-\tau) - x_1) + \alpha_4(y_4(t - \\
\tau) - x_4) - 2\alpha_5(y_5(t-\tau) - x_5), \\
u_3 = -(y_3(t-\tau) - x_3), \\
u_4 = -(y_4(t-\tau) - x_4) - \alpha_3(y_2(t-\tau) - x_2), \\
u_5 = -(y_5(t-\tau) - x_5) - \alpha_5(y_4(t-\tau) - x_4).
\end{gathered} \tag{20}$$

The parameters update law for the unknown 5-d hyperchaotic system is rewritten as

$$\begin{gathered}
\dot{\alpha}_1 = x_1e_1 + x_2e_2, \\
\dot{\alpha}_2 = x_1e_2, \\
\dot{\alpha}_3 = -x_2e_4, \\
\dot{\alpha}_4 = x_4e_1 + x_4e_2, \\
\dot{\alpha}_5 = -(x_1 + x_2 + x_4)e_5 - x_5e_1 - x_5e_2.
\end{gathered} \tag{21}$$

In fact, the synchronization error would be changed as

$$e_i = y_i(t-\tau) - x_i \; (i = 1 \ldots 5) \tag{22}$$

where τ is a time delay.

Time-delayed control for the unknown 5-d non-autonomous system can be written as

$$\begin{gathered}
u_1 = (\alpha_1 - 1)(y_1(t-\tau) - x_1) + \alpha_4(y_4(t-\tau) - x_4) - 2\alpha_5(y_5(t-\tau) - x_5) - \\
x_3(y_2(t-\tau) - x_2), \\
u_2 = (\alpha_1 - 1)(y_2(t-\tau) - x_2) + \left((1 - e^{-\lambda t})\alpha_2 - y_3\right)(y_1(t-\tau) - x_1) + \alpha_4(y_4(t - \\
\tau) - x_4) - 2\alpha_5(y_5(t-\tau) - x_5), \\
u_3 = -(y_3(t-\tau) - x_3), \\
u_4 = -(y_4(t-\tau) - x_4) - \left(1 - e^{-\lambda t}\right)\alpha_3(y_2(t-\tau) - x_2) \\
u_5 = -(y_5(t-\tau) - x_5) - \alpha_5(y_4(t-\tau) - x_4).
\end{gathered} \tag{23}$$

The parameters update law for the unknown 5-d non-autonomous system is defined as

$$\begin{gathered}
\dot{\alpha}_1 = x_1e_1 + x_2e_2, \\
\dot{\alpha}_2 = x_1e_2, \\
\dot{\alpha}_3 = -x_2e_4, \\
\dot{\alpha}_4 = x_4e_1 + x_4e_2, \\
\dot{\alpha}_5 = -(x_1 + x_2 + x_4)e_5 - x_5e_1 - x_5e_2
\end{gathered} \tag{24}$$

where $e_i = y_i(t-\tau) - x_i \; (i = 1 \ldots 5)$.

4 .Numerical simulations

The numerical simulation contains three main parts: the simulation results for an unknown hyperchaotic system, uncertain non-autonomous model, and implementation of delayed control input on both systems.

4.1.Results for stabilization of Uncertain hyperchaotic system

In this section, the results of adaptive synchronization appliance on unknown hyperchaotic Rikitake system are discussed. In this simulation, the parameters for the hyperchaotic system (1) are taken from reference No. [36].

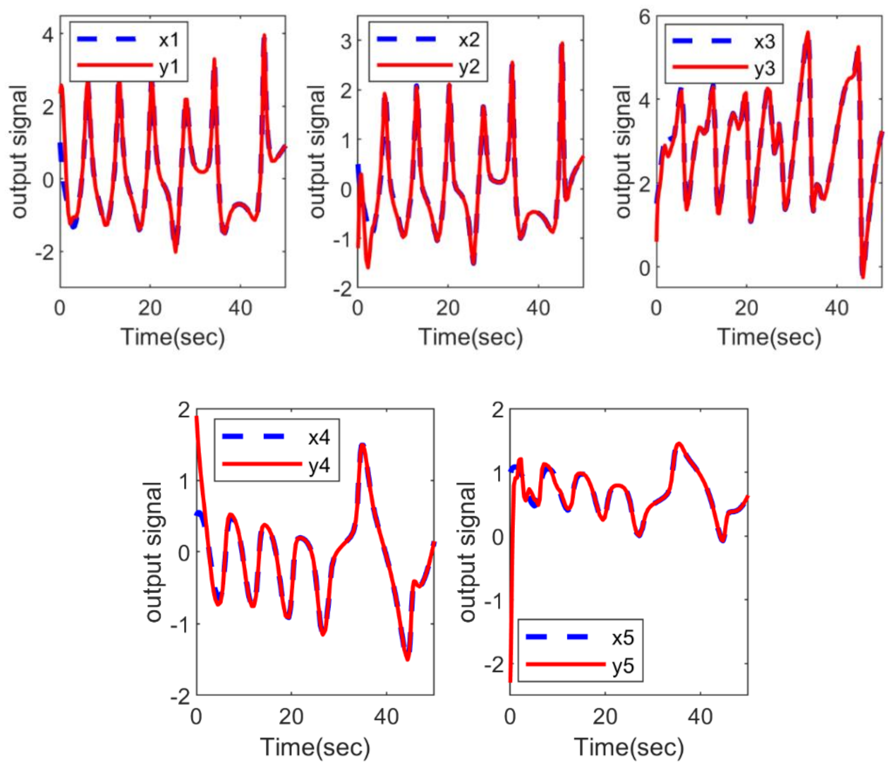

Fig. 3: Time-history for complete synchronization of the different state variables.

The parameters for the uncontrolled system (1) are considered as:

$$\theta_1 = 2, \theta_2 = 1.5, \theta_3 = 0.5, \theta_4 = 1.5, \theta_5 = 0.1.$$

The initial conditions for the driver system (1) are assumed as:

$$x_1(0) = 1, x_2(0) = 0.5, x_3(0) = 1.5, x_4(0) = 0.5, x_5(0) = 1.$$

The initial states of the controlled system (5) are taken as:

$$y_1(0) = 2.4, y_2(0) = -1.2\ y_3(0) = 0.6, y_4(0) = 1.9, y_5(0) = -2.3.$$

Respectively, the initial guesses for the uncertain parameters in the slave system (5) are taken as:

$$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 0.7, \alpha_4 = 1.1, \alpha_5 = 0.1.$$

The numerical simulations are created to illustrate the effectiveness of the proposed method.

Fig. **3** describes the adaptive synchronization of the specific output signal of the 5-D hyperchaotic systems (1) and (5). It is shown that there is a difference between the behavior of a derived system (1) and a controlled system (5) at first. However, the behavior will be completely the same after 15 seconds.

4.2.Results for stabilization for time-varying system

In this step, stabilization results of synchronization of the non-autonomous Rikitake system are shown. Fig. **4** denotes that this controller stabilizes the new system. As it is seen in Fig. **4**, the time for identification of the system and convergence of the state's increase.

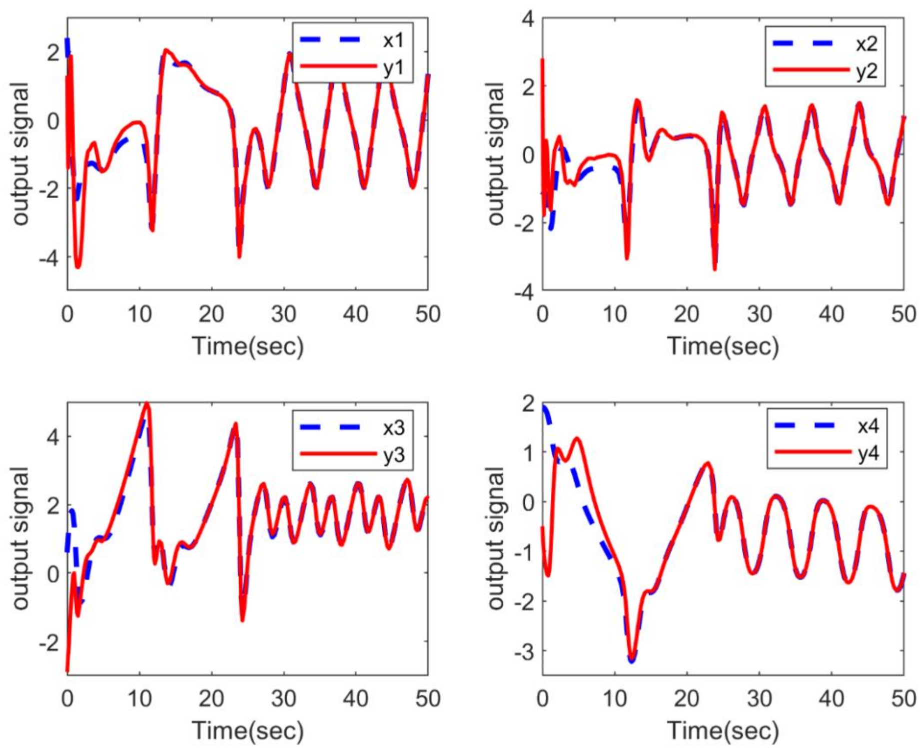

Fig. 4: Time-history for complete synchronization of the state variables of the non-autonomous system.

4.3.Results for stabilization of Rikitake and autonomous system with delayed control input

In this section, the results of stabilization when time-delayed control implement on both proposed systems are depicted. Fig. **5** indicates the synchronization of the output signals of the 5-D hyperchaotic systems (1) and (5) when the control input with delay is implemented to the system. It can be concluded that there is a little disturbance in synchronization and the behavior of the master and slave systems are not completely the same. However, the behavior of the unstable system is changed and somehow, it can be stated that the behavior of the system is now stable.

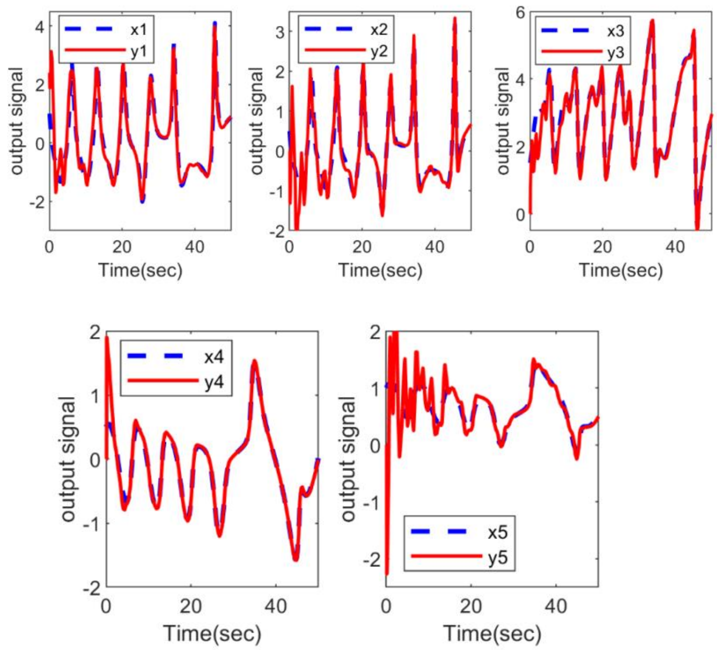

Fig. 5: Time-history for complete synchronization of the different state variables with time-delayed control input.

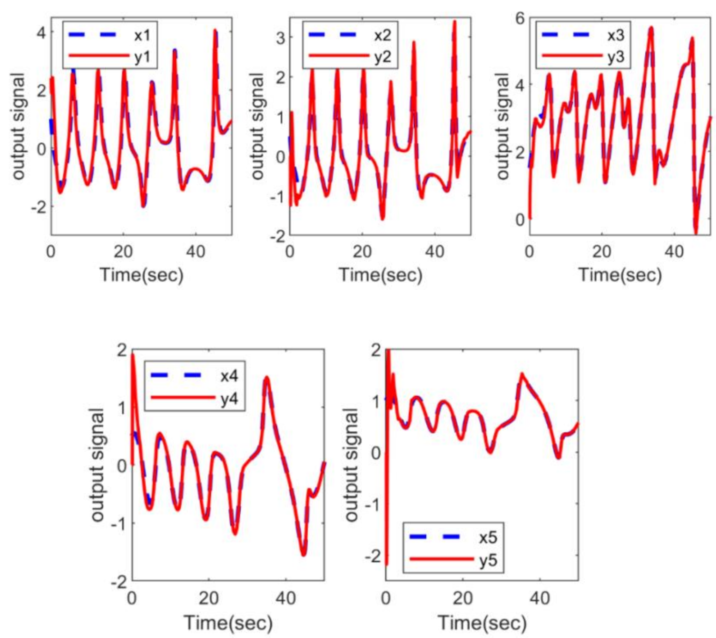

Fig. 6: Time-history for complete synchronization of the different state variables with time delayed control input and higher control again.

In order to completely transform the behavior of the system into stable behavior, the gain control of the system is slightly increased. Fig. **6** shows the synchronization of the output signals of the master and slave systems with the time-delayed control input when the control gains are higher than before. It demonstrates that the unstable behavior is completely changed to stable behavior in a short time.

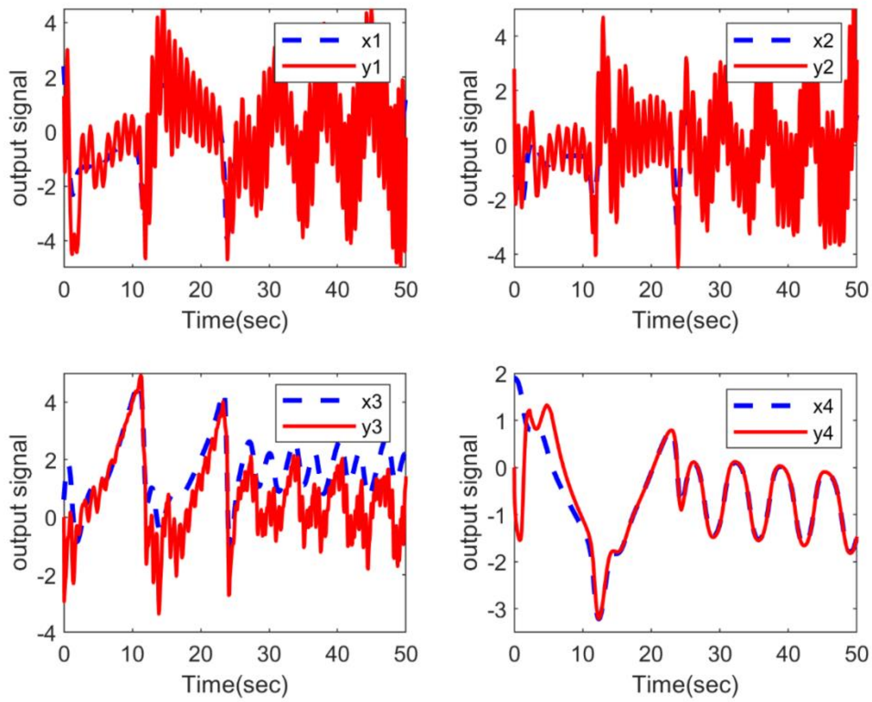

Fig.7: Time-history for complete synchronization of the state variables of non-autonomous system with time-delayed control input.

Fig.7 demonstrates the synchronization of the non-autonomous system with a delayed control input. The behavior of the system is changed, but the disturbance is too big in some states. Therefore, the controller is unable to stabilize the system.

By increasing the control gain, stabilization can be achieved. Fig. **8** indicates that the controller is able to transform the unstable behavior into a stable one. However, it can be seen from fig, there is a little disturbance in synchronization.

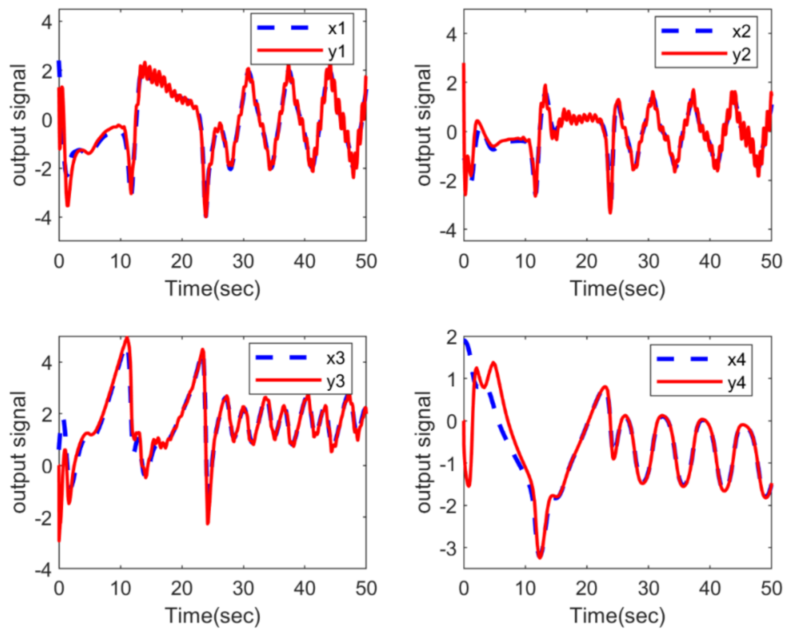

Fig. 8: Time-history for complete synchronization of the state variables of non-autonomous system with time-delayed control input and higher control gain.

Table 1. The results of adaptive synchronization for various chaotic systems.

Type of system	Applying adaptive law
Autonomous system	Complete synchronization after 15 s
Non-autonomous system	Complete synchronization after 17 s
Autonomous system with time-delayed control input	The synchronization is not completely achieved. However, the behavior of the master and slave system will be almost similar after 20 s.
Autonomous system with time-delayed control input and higher control again	Complete synchronization after 18 s
Non-autonomous system with time-delayed control input	Synchronization is not achieved. The system is unstable.
Non-autonomous system with time-delayed control input and higher control gain	The synchronization is not completely achieved and there is a little disturbance in. However, the behavior of the system is changed.

Table 1 is presented to outline the results of the synchronization of different systems and compare them in a better way.

It can be concluded that as the chaotic system becomes more complex and the controller becomes closer to reality, the adaptive control finds it difficult to synchronize the behavior of the master and slave systems. The robust or fuzzy controller could be able to solve this problem and decrease the time that it takes to achieve complete synchronization in a wide number of systems.

5. Conclusion

In this paper, the issue of the adaptive synchronization of uncertain 5-D hyper-chaotic Rikitake system with unknown system parameters was investigated in detail. Although the uncertain 5-D hyperchaotic Rikitake system has been examined by several scientists before, to the best knowledge of the authors, controlling such a system by using an adaptive synchronization has not been taken into consideration yet. Synchronization of 5-D hyper-chaotic Rikitake system could be achieved by this effective method and it could be adopted for a class of non-autonomous chaotic systems. In addition, the existence of a delay in adaptive synchronization input for both Rikitake and time-varying system was investigated and proved that the behavior of the systems could be changed from unstable to stable one by this strategy. In this article, numerical simulations were offered to explain the feasibility and performance of an identification method. Theresults concluded that the time evolutions for the variables of both systems with or without delay in control input would be the same in a few minutes. It should be noted that the behavior of the master system and slave system has been similar and there is no indication of the chaotic behavior in the system. In other words, the system profoundly transforms from chaotic into a periodic one after a short time and the stability of the system is guaranteed. The presented identification method can be employed practically for some complex nonlinear systems and help to deal with serious problems in this field.

References

[1] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, Physical Review Letters, 64 (1990) 821-824.

[2] B. Vaseghi, M.A. Pourmina, S. Mobayen, Finite-time chaos synchronization and its application in wireless sensor networks, Transactions of the Institute of Measurement and Control, 40 (2018) 3788-3799.

[3] A. Khan, S. Kumar, Measuring chaos and synchronization of chaotic satellite systems using sliding mode control, Optimal Control Applications and Methods, 39 (2018) 1597-1609.

[4] M. Bettayeb, U.M. Al–Saggaf, S. Djennoune, Single channel secure communication scheme based on synchronization of fractional-order chaotic Chua’s systems, Transactions of the Institute of Measurement and Control, 40 (2018) 3651-3664.

[5] L. Liu, G. Xie, R. Li, Synchronization stability analysis of medical cyber-physical cloud system considering multi-closed-loops, Journal of Circuits, Systems and Computers, 28 (2019) 1950198.

[6] A.T. Azar, S. Vaidyanathan, Computational intelligence applications in modeling and control, Springer, 2014.

[7] A.T. Azar, S. Vaidyanathan, Chaos modeling and control systems design, Springer, 2015.

[8] E.N. Lorenz, Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, 20 (1963) 130-141.

[9] O.E. Rössler, An equation for continuous chaos, Physics Letters A, 57 (1976) 397-398.

[10] G. Chen, T. Ueta, Yet another chaotic attractor, International Journal of Bifurcation and Chaos, 9 (1999) 1465-1466.

[11] H.-K. Chen, C.-I. Lee, Anti-control of chaos in rigid body motion, Chaos, Solitons & Fractals, 21 (2004) 957-965.

[12] G. Cai, Z. Tan, Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain systems, 1 (2007) 235-240.

[13] V. Sundarapandian, Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, 6 (2013) 45-52.

[14] A. Arneodo, P. Coullet, C. Tresser, Possible new strange attractors with spiral structure, Communications in Mathematical Physics, 79 (1981) 573-579.

[15] J.C. Sprott, Some simple chaotic flows, Physical Review E, 50 (1994) R647.

[16] V.-T. Pham, S. Vaidyanathan, C. Volos, S. Jafari, S.T. Kingni, A no-equilibrium hyperchaotic system with a cubic nonlinear term, Optik-International Journal for Light and Electron Optics, 127 (2016) 3259-3265.

[17] S. Vaidyanathan, C.K. Volos, V.-T. Pham, Analysis, Adaptive Control and Adaptive Synchronization of a Nine-Term Novel 3-D Chaotic System with Four Quadratic Nonlinearities and its Circuit Simulation, Journal of Engineering Science & Technology Review, 8 (2015).
[18] S. Vaidyanathan, C. Volos, Advances and Applications in Chaotic systems, Springer, 2016.
[19] S. Vaidyanathan, A.T. Azar, Adaptive backstepping control and synchronization of a novel 3-D jerk system with an exponential nonlinearity, in: Advances in chaos theory and intelligent control, Springer, 2016, pp. 249-274.
[20] S. Vaidyanathan, Hyperchaos, qualitative analysis, control and synchronisation of a ten-term 4-D hyperchaotic system with an exponential nonlinearity and three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 23 (2015) 380-392.
[21] V.-T. Pham, S. Vaidyanathan, C. Volos, S. Jafari, F.E. Alsaadi, F.E. Alsaadi, Chaos in a simple snap system with only one nonlinearity, its adaptive control and real circuit design, Archives of Control Sciences, 29 (2019).
[22] R. Behinfaraz, S. Ghaemi, S. Khanmohammadi, Adaptive synchronization of new fractional-order chaotic systems with fractional adaption laws based on risk analysis, Mathematical Methods in the Applied Sciences, 42 (2019) 1772-1785.
[23] S. Vaidyanathan, S. Sampath, Anti-synchronisation of identical chaotic systems via novel sliding control and its application to a novel chaotic system, International Journal of Modelling, Identification and Control, 27 (2017) 3-13.
[24] X. Chen, T. Huang, J. Cao, J.H. Park, J. Qiu, Finite-time multi-switching sliding mode synchronisation for multiple uncertain complex chaotic systems with network transmission mode, IET Control Theory & Applications, 13 (2019) 1246-1257.
[25] X. Yang, Q. Song, J. Cao, J. Lu, Synchronization of coupled Markovian reaction–diffusion neural networks with proportional delays via quantized control, IEEE transactions on neural networks and learning systems, 30 (2018) 951-958.
[26] I. Ahmad, A.B. Saaban, A.B. Ibrahim, M. Shahzad, Global chaos synchronization of new chaotic system using linear active control, Complexity, 21 (2015) 379-386.
[27] A. Kazemy, M. Farrokhi, Synchronization of chaotic Lur'e systems with state and transmission line time delay: a linear matrix inequality approach, Transactions of the Institute of Measurement and Control, 39 (2017) 1703-1709.
[28] Y.-J. Liu, S. Tong, Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems, Automatica, 76 (2017) 143-152.
[29] S.-Y. Li, M.A.B. Hernández, Robust synchronization of chaotic systems with novel fuzzy rule-based controllers, Information Sciences, 481 (2019) 604-615.
[30] W. Ao, T. Ma, R.-V. Sanchez, H. Gan, Finite-time and fixed-time impulsive synchronization of chaotic systems, Journal of the Franklin Institute, (2019).
[31] Y. Wang, D. Tong, Q. Chen, W. Zhou, Exponential synchronization of chaotic systems with stochastic perturbations via quantized feedback control, Circuits, Systems, and Signal Processing, 39 (2020) 474-491.
[32] M. Asadollahi, A.R. Ghiasi, M.A. Badamchizadeh, Adaptive synchronization of chaotic systems with hysteresis quantizer input, ISA transactions, 98 (2020) 137-148.
[33] I. Ahmad, M. Shafiq, Oscillation free robust adaptive synchronization of chaotic systems with parametric uncertainties, Transactions of the Institute of Measurement and Control, (2020) 0142331220903668.
[34] T. Rikitake, Oscillations of a system of disk dynamos, in: Mathematical Proceedings of the Cambridge Philosophical Society, Cambridge University Press, 1958, pp. 89-105.
[35] X. Yu, Y. Song, Chaos synchronization via controlling partial state of chaotic systems, International Journal of Bifurcation and Chaos, 11 (2001) 1737-1741.
[36] S. Vaidyanathan, V.-T. Pham, C. Volos, A 5-D hyperchaotic Rikitake dynamo system with hidden attractors, The European Physical Journal Special Topics, 224 (2015) 1575-1592.