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# Vibration damping of piezo actuating composite beams based on the multi-objective genetic algorithm

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**ABSTRACT**

In this work, a multi-objective optimization process based on the genetic algorithm is employed to damp the vibrations of a piezo actuating composite beam. A new mathematical model for the control effort is proposed and optimized with two objective functions. Conflicting objectives are considered as the displacement of the beam and the second derivative of the control voltage. The coefficients of the proposed control voltage model are regarded as the design variables for this optimization process. The corresponding Pareto front represents non-dominated optimum solutions with different choices to designers. The time behaviors of displacement, velocity and acceleration as well as the related control effort at the midpoint of the beam for three optimum design points are illustrated. The simulation of the time responses of a selected optimum point exhibits the advantage of the planned optimum strategy with regard to those stated in some research such as the cases used in the maximum principle for the same structure.



## 1.Introduction

In the recent years, many new approaches have been developed to suppress the oscillation of the structures. Among them, applying the smart materials, such as piezoelectrics, is one of the most

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prominent solutions for the structures having external disturbance loads. In fact, the vibration control problem of deformable bodies using the piezoelectric materials is an emergent research subject with various applications. As an original work, Banks *et al*. introduced the mathematical modeling and control approaches of the smart material structures [1]. Furthermore, Preumont represented an outline of piezoelectrics and their vibration control methods [2]. Tzou and Fu discussed sensing and control effectiveness of the segmented sensors and actuators. They derived mathematical formulations for a plate having a single-piece symmetrically distributed and quarterly segmented-distributed sensors and presented the related analytical solutions. Accordance to the analytical solutions, they proved that the single-piece symmetrically spread sensor layers are deficient for antisymmetrical modes of the plate. Besides, they indicated that the quarterly segmented spread sensors could handle nearly all of the natural modes [3]. Lee *et al.* developed the theory of the modeling of the electromechanical (actuating) and mechanoelectrical (sensing) behavior for the laminated piezoelectric plates [4]. They found that the reciprocal relationship of the piezoelectric sensors and actuators is a generic feature of all piezoelectric laminates. Gaudenzi *et al*. numerically simulated and experimentally analyzed the vibration attenuation effects of active cantilever beams and successfully controlled the positions and velocities of the regarded points[5] . Ray proposed an optimum method for stabilization of a laminated plate having piezoelectric sensors and actuator layers [6]. Qiu *et al*. utilized sensors and actuators constructed from piezoelectric ceramic patches to handle the oscillation of an elastic plate with the clamped boundary conditions [7]. Gardonio and Elliott investigated the velocity feedback control of a vibratory beam utilizing its modal responses, sensors and actuators [8]. Inman introduced a methodology for the active modal control of the structures made from piezo materials [9]. Optimum design of a boundary controller for dynamical oscillation of piezo actuating composite beams having micro sizes was studied by Sadek *et al*. [10]. Baz investigated boundary control and vibration damping of beams by active constrained layers [11]. The vibration damping of plates with rectangular shapes was presented via a special type of modal sensors and the active boundary control methodology by Kaizuka and Tanaka [12]. Hwu *et al*. investigated smart cantilevered composite sandwich beams covered by piezoelectric sensors and actuators[13]. Baillargeon and Vel experimentally and numerically studied the oscillation handling of smart structures utilizing piezoelectric shear actuators[14]. Stavroulakis *et al*. offered a methodology for oscillation damping of a beam covered by sensors and actuators made from piezoelectric materials and introduced its applications [15]. Vahdati and Heidari applied a three-layer piezoelectric beam with cantilever boundary conditions for a standard single-pumper and provided a fluid support with an adjustable level frequency [16]. Raja *et al*. used piezoelectric multilayered actuators for the flutter control of a composite plate[17]. In a similar work, Han *et al*. utilized piezoelectric actuation for active vibration damping of a lifting surface via modern control theories[18]. Bruant and Proslier implemented active tools for the vibration handling of a piezoelectric beam having functionally graded materials [19]. They illustrated the effect of the properties of the used material on the control system characteristics and proved the robustness of their proposed strategy. Kucuk *et al*. derived the maximum principle approach for the optimum design of a controller for a beam having especial dampers and piezoelectric patch actuators with external excitations [20]. A competent model was proposed by Chandrasekhar and Donthireddy to reveal the dynamical reaction of a special kind of piezoelectric composites [21]. Furthermore, they demonstrated the capability of the feedback control for the active suppression of the vibrations of the beams constructed from the laminated composites. Hsu *et al.* employed actuators with layered modal sensors for controlling beam vibrations [22]. Kim designed and implemented a robust control technique in order to decline the oscillations of a vibrant structure [23]. Nehru studied the oscillations of piezoelectric layers and focused on the vibration problem of an infinite piezo-laminated multilayer hollow cylinder[24] . Xue and Tang developed a scheme derived from an integral sliding mode approach to control the oscillation of a piezoelectric nonlinear rotating beam formulated by the Euler-Bernoulli assumptions[25]. Constructing several kinds of piezo actuators and their descriptions were investigated by Sahoo and Panda [26]. Zenz and Humer discussed the stability of beam-type structures under a compressive force and increased the critical buckling load by piezoelectric transducers [27]. Shirazi et al. introduced a novel methodology for the tracking control of mathematically modeled micro-cantilever beams based on the Timoshenko theory through the piezoelectric actuators [28]. Lara *et al*. investigated the oscillation suppression of piezo actuation beams utilizing an optimal boundary control theory [29]. Sloss *et al*. applied the maximum principle theory for the optimum design of a controller to damp the oscillations of structures and employed their strategy to suppress the vibrations of beams [30]. Collect *et al*. derived the governing equations of a micro-piezo-beam with the cantilever boundary condition and analyzed its active damping [31]. Ha and Hale described the properties of different grades of the piezoelectric ceramic powder for manufacturing piezoelectric paint [32]. Parameswaran and Gangadharan focused on the oscillation damping of a cantilever beam mathematically modelled in the parametric domain at its first resonant frequency. They utilized the finite element methods for modeling their intelligent system and demonstrated the correctness of the simulation results through comparison with those of experimental [33]. Xu and Du represented the dynamical relations for computing the performance of a motor vibrator and investigated its natural frequencies and shape modes [34]. Kim used an equivalent electrical circuit model and developed an algorithm to efficiently identify the relevant circuit parameters of arbitrarily-shaped cantilevered piezoelectric energy harvesters [35]. Rao and Muralidhara employed a piezo micro actuator based on a system of hydraulic displacement amplification to experimentally investigate its properties [36]. Chung *et al*. forecasted the dynamical properties of hydraulic components of a piezo injector having the bypass-circuit via a full-circuit numerical model and using a new developed code. Furthermore, they confirmed the proposed model by comparison with the obtained experimental results [37]. Xu and Xing investigated the piezo-ceramic bimorph subjected to the excitation signals and found the relations of the voltage response for an inertial piezoelectric rotary motor [38]. Kwon *et al*. analyzed the electromechanical characteristics of various types and sizes of piezoelectric powder thick films and constructed an optimum kind of them[39]. In order to power the wireless sensor nodes in a smart grid, Chen *et al*. designed an energy harvester associated with piezoelectric materials and electromagnetic mechanisms[40]. Atai and Lak analytically investigated the mechanical and electrical characteristics of a piezoelectric hollow sphere constructed from the functionally graded materials [41].

In this research, in order to damp the vibrations of the piezo actuating composite beam, a mathematical model based on its dynamical differential equations is proposed for the control voltage. The genetic algorithm optimization is implemented to find the parameters of the considered model via multi-objective optimum design concepts. Conflicting objectives are considered as the displacement of the beam and the second derivative of the control voltage. The coefficients of the proposed control voltage model are regarded as the design variables. The results of this straightforward approach are compared with those of other complicated methods, and the superiority of the proposed approach is proved. It is noticeable that the proposed strategy in this work for the vibration damping of the piezo actuating composite beam is substantially different with the introduced technique in Reference [10]. In this reference, Sadek *et al*. derived an optimal control law using a maximum principle based on a Hamiltonain expressed in terms of an adjoint variable as well as admissible control functions. The explicit solution of the problem was developed using eigenfunction expansions of the state and adjoint variables linked by the terminal conditions. Whereas, in the current research, a mathematical model based on the vibration behavior of the system is optimized using the multi-objective genetic algorithm. Moreover, the numerical results of the two approaches are compared with together, and the capability of the proposed scheme is given.

The rest of this paper is organized as follows. Section 2 represents the governing equations of the considered piezo compost beam. Section 3 demonstrates the concepts of the genetic algorithm and multi-objective optimization as well as the related objective functions. The simulation results and analysis are discussed in Section 4. Finally, Section 5 concludes the paper.

## 2.Dynamical differential equations of the piezo actuating composite beam

Piezoelectric actuators as effective stabilizer devices have been broadly implemented for active handling of structural vibrations. Further, Piezo materials are commonly integrated with some structural components such as distributed sensors and active dampers of oscillations. Hence, the active control of the fluctuations appeared in the structures, such as beams, plates, shells etc. using such smart materials has numerous applications, especially in aerospace engineering, flexible manipulators, antennas, earthquake resistant structures and so on. Preumont [1] and Banks *et al*. [2] precisely investigated the applications of the smart materials for vibration damping of the active structures.

A piezo actuating composite beam having length $\Delta$, width $w$ and thickness $t_b$ is considered with two layers of piezoelectric materials (up and down) having thickness $t_p$ (Fig. 1). The equations of motion for the beam based on the Euler–Bernoulli theorem could be described as follows [31].

$$\vartheta\psi_{\tau\tau} + EI\psi_{\delta\delta\delta\delta} = 0,, \quad 0 < \delta < \Delta \;\; , \;\; 0 < \tau < \tau_f \tag{1}$$

where, $\tau$ denotes time, and $\delta$ signifies the position of the considered point with respect to the following boundary conditions (simply supported):

$$\begin{gathered} \psi(0,\tau) = 0 \text{ and } \psi(\Delta,\tau) = 0 \\ \psi_{\delta\delta}(0,\tau) = \frac{G}{EI}f(\tau) \quad \text{and} \quad \psi_{\delta\delta}(\Delta,\tau) = \frac{G}{EI}f(\tau) \end{gathered} \tag{2}$$

and initial conditions:

$$\psi(\delta,0) = \psi_0(\delta) \quad \text{and} \quad \psi_{\tau}(\delta,0) = \psi_1(\delta) \tag{3}$$

where,

$$\begin{gathered} EI = \Omega - \frac{\beta^2}{\alpha}, \qquad \alpha = \left(E_b t_b + E_p t_p\right)w \\ \beta = (\frac{E_p - E_b}{2})t_p t_b w \\ \Omega = (\frac{E_b t_b(t_b^2+3t_p^2) - E_p t_p(t_p^2+3t_b^2)}{12})w,\; G = \left(\frac{t_b}{2} - \frac{\beta}{\alpha}\right)w\mu,\; \mu = \kappa E_p \end{gathered} \tag{4}$$

which, $\vartheta$, $EI$, $E_p$ and $E_b$ represent the length density, bending stiffness, Young's modulus of the piezoe layers and Young's modulus of the elastic beam, respectively. Moreover, $\kappa$ demonstrates a constant related to the value of the piezoelectric parameter for the actuator, $\psi(\delta,\tau)$ shows the beam displacement in the $\sigma$ direction, and $f(\tau)$ exhibits the implemented voltage for controlling the vibration of the beam [31]. For simplicity, the following non-dimensional variables are defined.

$$\Psi(\Gamma,\mathrm{T}) = \frac{\psi(\delta,\tau)}{\Delta}, \Gamma = \frac{\delta}{\Delta}, F(\mathrm{T}) = \frac{G}{\Delta EI} f(\tau)$$
$$\mathrm{T} = \frac{\tau}{\Delta^2}\sqrt{\frac{EI}{\vartheta}}, T_f = \frac{\tau_f}{\Delta^2}\sqrt{\frac{EI}{\vartheta}}, \tag{5}$$

where, $\Psi(\Gamma,\mathrm{T})$, $\Gamma$, $F(\mathrm{T})$ and $\mathrm{T}$ are regarded as the dimensionless displacement, position, control voltage and time variables, respectively. The non-dimensional dynamical differential equation would be obtained by substituting Eq. (5) into Eq. (1).

$$\Psi_{\mathrm{TT}} + \Psi_{\Gamma\Gamma\Gamma\Gamma} = 0, \quad 0 < \Gamma < 1, \quad 0 < \mathrm{T} < \mathrm{T}_f \tag{6}$$

with the following boundary and initial conditions.

$$\Psi(0,\mathrm{T}) = 0 \text{ and } \Psi_{\Gamma\Gamma}(0,\mathrm{T}) = F(\mathrm{T})$$
$$\Psi(1,\mathrm{T}) = 0 \text{ and } \Psi_{\Gamma\Gamma}(1,\mathrm{T}) = F(\mathrm{T}) \tag{7}$$

$$\Psi(\Gamma,0) = \Psi_0(\Gamma) \text{ and } \Psi_{\mathrm{T}}(\Gamma,0) = \Psi_1(\Gamma) \tag{8}$$

The mentioned non-homogenous boundary conditions in Eq. (7) may be converted to the homogenous forms via a new variable illustrated as follows.

$$\bar{\Psi}(\Gamma,\mathrm{T}) = \Psi(\Gamma,\mathrm{T}) - \frac{1}{2}(\Gamma^2 - \Gamma)F(\mathrm{T}) \tag{9}$$

Then, Eq. (6) becomes:

$$\bar{\Psi}_{\mathrm{TT}}(\Gamma,\mathrm{T}) + \bar{\Psi}_{\Gamma\Gamma\Gamma\Gamma}(\Gamma,\mathrm{T}) = -\frac{1}{2}(\Gamma^2 - \Gamma)\ddot{F}(\mathrm{T})$$
$$0 < \Gamma < 1 \quad , \quad 0 < \mathrm{T} < \mathrm{T}_f \tag{10}$$

including the following boundary and initial conditions.

$$\bar{\Psi}(0,\mathrm{T}) = 0 \text{ and } \bar{\Psi}(1,\mathrm{T}) = 0$$
$$\bar{\Psi}_{\Gamma\Gamma}(0,\mathrm{T}) = 0 \text{ and } \bar{\Psi}_{\Gamma\Gamma}(1,\mathrm{T}) = 0 \tag{11}$$

$$\bar{\Psi}(\Gamma,0) = \Psi_0(\Gamma) - \frac{1}{2}(\Gamma^2 - \Gamma)F(0)$$
$$\bar{\Psi}_{\mathrm{T}}(\Gamma,0) = \Psi_1(\Gamma) - \frac{1}{2}(\Gamma^2 - \Gamma)\dot{F}(0) \tag{12}$$

If the sine Fourier series are utilized to achieve the answer of Eq. (10), then

$$\bar{\Psi}(\Gamma,\mathrm{T}) = \sum_{i=1}^{N} Z_i(T)\sin(w_i\Gamma), \qquad w_i = i\pi \tag{13}$$

where, $N$ is the term number of the sine Fourier series. Replacing Eq. (13) in Eq. (10) leads to Eq. (14).

$$\sum_{i=1}^{N}\left\{\ddot{Z}_i(T)+w_i^4 Z_i(T)\right\}\sin(w_i\Gamma)=-\frac{1}{2}(\Gamma^2-\Gamma)\ddot{F}(\mathrm{T}) \tag{14}$$

Eventually, the orthogonality characteristic of the sine Fourier series causes that Eq. (14) changes to Eq. (15).

$$\ddot{Z}_i(T)+w_i^4 Z_i(T)=-2\int_0^1\frac{1}{2}(\Gamma^2-\Gamma)\sin(w_i\Gamma)d\,\Gamma\ddot{F}(\mathrm{T}) \tag{15}$$

In order to control and damp the vibrations of the piezo actuating composite beam, the effects of the piezo actuators emerged in the boundaries $\Gamma=0$ and $\Gamma=1$ are appropriately employed. Hence, the following formulation is proposed for the control voltage function $F(\mathrm{T})$ regarding the harmonically behavior of the beam.

$$F(T)=\text{Error! Bookmark not defined.}\,dT \tag{16}$$

where, design vector $\boldsymbol{C}=(c_1,c_2,c_3,c_4,c_5)$ would be optimally determined by the multi-objective genetic algorithm optimization. Eqs. (15) and (16) are simultaneously solved by employing Runge-Kutta approach in the environment of MALAB software. It is noticeable that after damping the vibrations of the beam, the exerted force should be removed from the structure in order to prevent the un-desirable oscillations.

## 3. Genetic algorithm optimization

The genetic algorithm is known as a probabilistic global search approach based on the biological evolution and modification of a population of individual solutions. The standard structure of the genetic algorithm could be mentioned as the following steps [42]:

a. Create the initial population of individuals (chromosomes).
b. Evaluate the fitness of chromosomes by using the defined objective functions.
c. Select some individuals to reproduce the new population by using the genetic operators (reproduction, crossover and mutation).
d. Iterate steps b-d until a stopping criterion is satisfied.

In this paper, the conflicting objective functions ($OF_1$ and $OF_2$) are regarded as follows:

$$OF_1=\int_0^{\mathrm{T}_f}|Z_i(\mathrm{T})|\,d\Gamma \text{ and } OF_2=\int_0^{\mathrm{T}_f}\left|\ddot{F}(\mathrm{T})\right|d\Gamma \tag{17}$$

which, $T_f$ is the final time. $Z_i(\mathrm{T})$ and $\ddot{F}(\mathrm{T})$ describe the displacement of the beam and the second derivative of the control voltage, respectively. Here, $T_f=10, i=1,2,\dots,N$ and $N=1$. Furthermore, the constant parameters of the proposed model for the control voltage function are considered as the design variable of these objectives. Finally, the configuration of the genetic algorithm is set as the values given in Table 1.

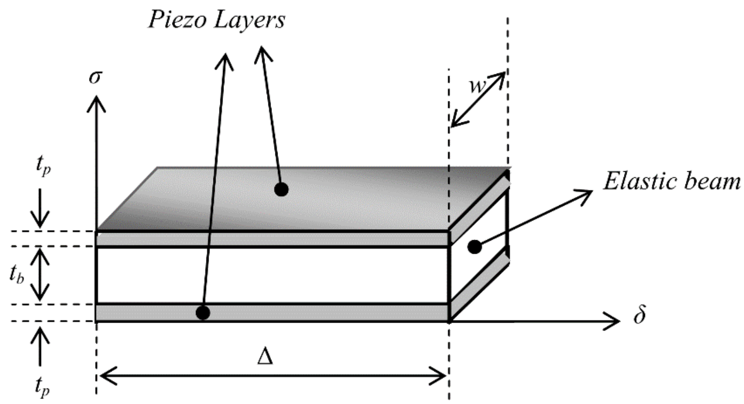


**Fig. 1**. Elastic beam with two layers of piezoelectric materials.

**Table 1**. Genetic algorithm configurations.

| | |
|---|---|
| Crossover fraction | 0.8 |
| Population size | 200 |
| Selection function | Tournament |
| Mutation function | Constraint dependent |
| Crossover function | Intermediate |
| Stopping criteria | Maximum iteration 420 |

**Table 2**. Objective functions and related design variables for the selected points displayed in Fig. 2.

| Point | Point A | Point B | Point C |
|---|---|---|---|
| $OF_1$ | 0.09511 | 0.20368 | 0.643773 |
| $OF_2$ | 0.45338 | 0.10406 | $3.34\times10^{-5}$ |
| $c_1$ | 11.12051 | 0.38655 | $-5.49\times10^{-4}$ |
| $c_2$ | -7.58311 | -0.33423 | $5.97\times10^{-4}$ |
| $c_3$ | -0.16481 | -0.11534 | -0.10877 |
| $c_4$ | -6.02013 | -3.55309 | $1.21\times10^{-4}$ |
| $c_5$ | -1.28276 | -0.28965 | 0.12069 |

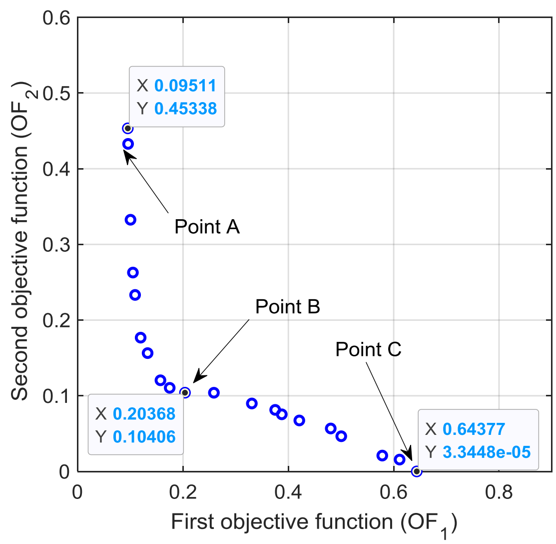


**Fig.2**. The obtained Pareto front for the proposed model for the vibration damping of the piezo actuating composite beam using the multi-objective genetic algorithm.

## 4. Results and discussion

In this section, the optimization of the proposed model is studied in order to damp the vibration of the considered beam with having the minimum value of the control voltage applied for the piezo actuators. For this purpose, the multi-objective genetic algorithm is employed to find the components of the design vector $\boldsymbol{C} = (c_1, c_2, c_3, c_4, c_5)$ with regard to two objective functions regarded as the second derivative of the control voltage and the displacement of the beam. The Pareto front obtained from this optimization process related to the control model of the piezo actuating composite beam with regard to the conflicting objective functions is depicted in Fig. 2. In this diagram, point A stands for the maximum second derivative of the control voltage and the minimum displacement of the beam. However, point C represents the maximum displacement of the beam and the minimum second derivative of the control voltage. The selected point B shown in Fig. 2 is a non-dominated optimum design point which stands for a good situation that satisfies two conflicting objectives in comparison with the other points. The values of the objective functions and corresponding design variables for the optimum points illustrated in the obtained Pareto front are given in Table 2. The time behavior of displacement, velocity and acceleration as well as the related control effort at the midpoint of the beam for these optimum design points are illustrated in Fig. 3, correspondingly. In Fig. 4, the time responses of the system states obtained by this research related to Point B are compared with those of the uncontrolled beam and method reported in [10]. The simulations and comparisons depict that the proposed mathematical model optimized by the multi-objective genetic optimization algorithm could display less overshoots and shorter setting times and is a valid method to damp the beam vibrations.

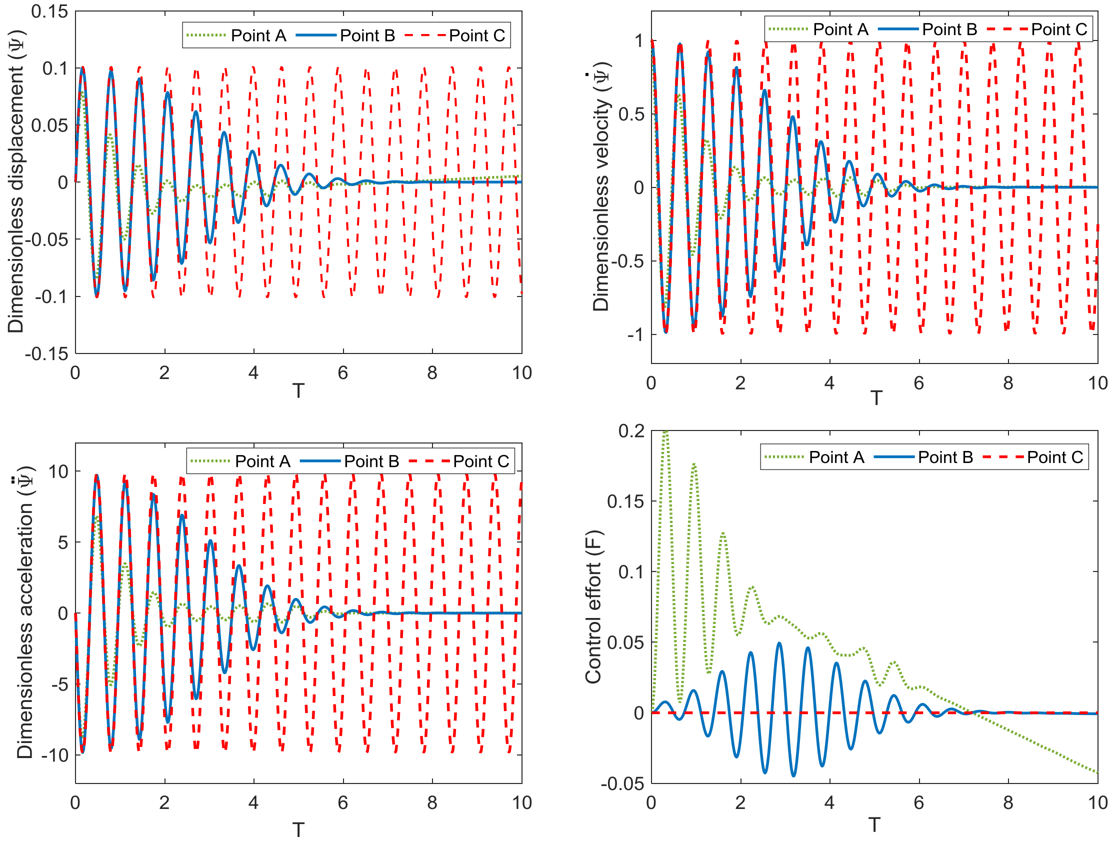


**Fig. 3**. Dimensionless displacement, velocity, acceleration and control effort at the midpoint of the piezo beam for the optimum design points depicted in the Pareto front.

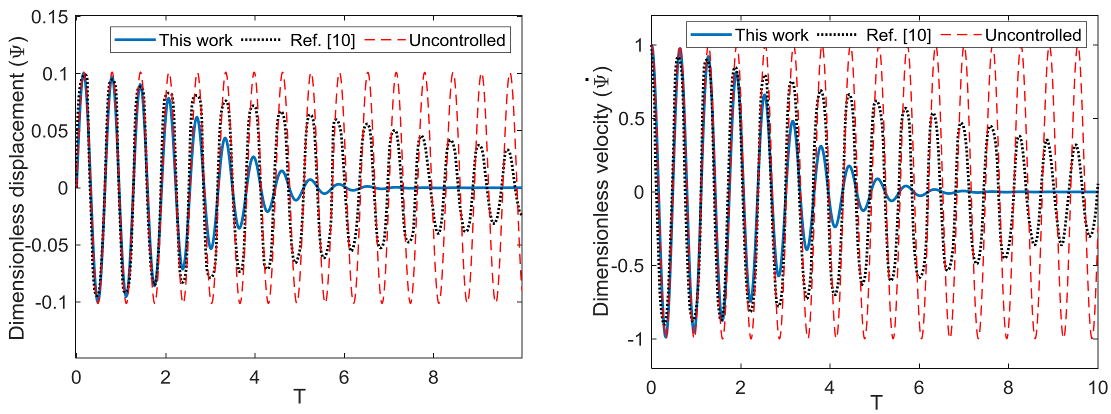


**Fig. 4**. Time responses of the states at the midpoint of the piezo beam for the suggested optimum design point B and the model proposed in Ref. [10].

## 5. Conclusion

This research work studied the vibration damping of Piezo actuating beams through multi-objective optimization algorithms. The governing dynamical differential equations of the piezo actuating composite beam was derived using Bernoulli theorem, and their dimensionless forms were presented. Next, a mathematical model was introduced for the control voltage function with regard to the harmonically behavior of the considered beam. Then, the multi-objective genetic algorithm was utilized for the optimum design of this control model with respect to two conflict objective functions, i.e. the displacement of the beam and the second derivative of the control voltage. The achieved Pareto front of this multi-criteria problem was illustrated, and the design variables and objective functions corresponding to three non-dominated optimum design points were presented. Using the proposed control model, the beam vibrations were damped in an acceptable level with the minimum voltage values applied on the piezo actuators. Finally, the feasibility and efficiency of the introduced methodology were assessed by comparisons of the time responses of the displacements and velocities of the beam for the proposed method and other reported approaches.

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