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Damage diagnosis of bridge girders via modal flexibility deflections, inertia force vectors, and strain energies

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This paper presents a new method for identifying damage in bridge girders based on modal flexibility deflections. Using modal inertia forces for spatial load distribution vector, a new approach for obtaining modal flexibility deflections is presented. A two-stage damage identification method is then proposed for beamlike structures using these flexibility deflections. In damage severity estimations, the concepts of static beam deflections are utilized. The abilities of the proposed method are demonstrated using experimental modal data of two full-scale bridge girders. Four damage scenarios of a steel plate girder and the surface damages in a pre-stressed concrete box girder are selected and discussed. The influence of uniform load surface and the number of vibration modes on damage patterns are studied. The results show that the localization resolution declines 15%-30% when applying the uniform load surface method, and the first two vibration modes are very influential on the damage patterns. In most cases, the predictions based on the flexibility deflections due to the modal inertia forces are found to closely (above 90%) agree with the actual damage of the bridge girders. Finally, the performance of the proposed method in the presence of modelling errors and noisy data is investigated. The results indicate that the proposed method is efficient in damage identification of beam-like structures.

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1. Introduction

The infrastructures such as bridges play a key role in the developments of urban transportation. During the service life of the bridges, the strength and stiffness, reduce due to the deterioration, natural, and service loads. The cracks and lack of stiffness may lead to catastrophic failures. Thus, numerous damage identification methods have been developed for locating and quantifying the damaged area of the bridge structures [1].

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Among these methods, the vibration-based identifying approaches which rely on the dynamic response of the entire structure are more effective [2]. A wide range of damage indices based on the dynamic response of the structure, such as modal strains, strain energy, and flexibility matrices, have been proposed [2, 3]. The flexibility-based identification methods have advantages in structural health monitoring. Using the first few modes of the structure, the modal flexibility can be calculated easily and does not require any numerical procedures. Though the formulation of the flexibility matrix is approximate, it is found that this approximation yields promising results [4, 5]. The components of the modal flexibility matrices are more sensitive to damage than the basic modal parameters such as natural frequencies and mode shapes [4].

The damage index can be computed by comparing the flexibility matrices in the pre-and post-damaged states. Pandey and Biswas [6] were the pioneers in the flexibility-based identification methods. They presented a level II damage identification technique based on the flexibility changes in the structure. They computed the maximum absolute value of flexibility changes in the pre- and post-damaged states. Utilizing these maximum values, damage existence and location are determined. After Pandey and Biswas [6], for three decades, considerable efforts were focused on developing damage identification based on modal flexibility [2]. Bernal [7] proposed a damage localization method based on changes in measured flexibility that was general with a consistent theoretical base. Bernal’s approach was called DLV or damage locating vectors. These vectors define a basis for the null space of the change in flexibility [7, 8]. Zonta et al. [9] estimated the flexibility matrix in a strain coordinate system. In the strain coordinate system, the strains are substituted for displacements and all calculations are performed in the strain space. This matrix is referred to as the strain-flexibility matrix, and the changes in each of its diagonal elements can be regarded as damage indices.

It has been found that the structural deflections computed from modal flexibility are very effective for damage identification. These deflections are recognized as flexibility deflections in the literature. The flexibility deflections can be assumed as the first Ritz vectors of the structures. Sohn and Law [10] used the extracted Ritz vectors for damage detection of a grid-type bridge model. According to Nour-Omid and Clough [11], the response quantities can be approximated more effectively by a smaller number of the Ritz vectors than the modal vectors. In addition, the components of the modal flexibility-based deflections or the first Ritz vectors extracted from a uniform vector load are less sensitive to the noise and errors due to modal truncations [12, 13]. The prominent features of the flexibility- deflections have led to development of effective damage detection methods for bridges [14-16] and building structures [12, 17, 18].

It should be mentioned that selection of the load vector for computing modal flexibility deflections plays an essential role in identify damaged areas in the structure. Special loads, which are interpreted as inspection loads, are utilized to estimate the modal flexibility deflections. The inspection loads usually depend on the type of structure under consideration. For damage identification of shear buildings, a special kind of inspection load is applied to compute the inter-story drifts. This inspection load is recognized as a positive shear inspection load (PSIL) [19]. Different forms of PSIL loads can be selected for damage identification [15]. The unit vector loads are the most widely used load inspection schemes [4, 20]. The uniform load surface method was developed based on the modal flexibility deflections due to the unit load vectors [16, 21, 22]. Koo et al. [12] developed a flexibility-deflection method to estimate deflection method to estimate damage severity in shear buildings. They utilized the changes of drifts in shear buildings for a damage severity estimation index. The proposed method proposed by Koo et al. [12], has practical advantages with no complicated optimization procedures. Therefore, this method was applied and extended in several research studies. Sung et al. [23] extended the proposed method by Koo et al. [12] for cantilever beam-type structures such as high-rise apartment buildings. Bernagozzi et al. [4] recommended the use of new PSIL loads and the concepts proposed by Koo et al. [12] for damage identification of a 3D frame structure. The inspection loads proposed by Bernagozzi et al. [4] were more complex than the uniform translational loads and, without any optimization tools, provided the possibility for damage severity estimation. Zare Hosseinzadeh et al. [24], used flexibility deflection concepts presented by Koo et al. [12] and introduced the damage diagnosis of 3D frames as an optimization problem. According to Zare Hosseinzadeh et al. [24], the complexities of the static displacement measurements, can be overcome by flexibility deflections. Le [25] provided an effective damage identification method based on flexibility deflections without time-consuming optimization algorithms. Le et al. [20] proposed a damage severity estimation method for simply supported beams based on the flexibility deflections. Their method has limitations for applying to beams with different boundary conditions and arbitrary loading.

Various aspects of the flexibility-deflection identification methods are still not thoroughly investigated. Some of these methods include difficulties associated with their implementation and their fidelity in the actual bridge structures [26]. These theoretical and practical problems are supplemented by many scientific issues. This paper tries to review the flexibility-deflection-based methods and find an efficient and practical damage identification method for bridge girders. Despite all the benefits of flexibility-deflection-based methods for damage locating, most of these methods are not capable enough of estimating the damage severity without a finite element model and time-consuming optimization procedures. The appropriate strategy for selecting the spatial load distribution vector and damage quantification without time-consuming numerical updating and optimization procedures are open challenges in this area that are investigated in this paper.

The main objective of this paper is to develop a flexibility-deflection damage identification method for damage localization and severity estimation in beam-like structures without complicated updating procedures and finite element models. For this purpose, a new method is developed based on the vector of modal inertia forces and modal flexibly matrices. The results show that the flexibility deflections that are obtained by modal inertia forces have prominent features in damage localization and severity estimations. The results of the proposed method for damage identification of progressive damage scenarios in the I-40 Bridge are compared to those obtained by uniform load surface and the classical Pandey and Biswas [6] method. The new approach is superior for damage localization, with a confidence level of 97.7%. The feasibility of the new method is also examined by experimental modal data of a pre-stressed concrete box girder. The very low-level surface damaged areas on the top of the concrete girder are identified by the proposed flexibility-deflection method. The changes of the strains of the flexibility-deflections due to the modal inertia forces yield promising results to show the damaged areas in concrete girders precisely with a higher confidence level. The performance of the proposed method in the presence of modelling errors and noisy data is investigated. The results of this study indicate that the new flexibility-based identification method provides a feasible estimate of damage location and severity in bridge girders.

2. Basic concepts of experimental modal flexibility

The components of the flexibility matrices are deformations corresponding to static forces of unit magnitude acting at the coordinates of the structure. In many practical applications, it is desirable to measure the displacement field with sufficient accuracy. Because of an impractical proposition, one is forced to synthesize the flexibility from measured vibration modes [7, 8]. The modal flexibility matrix can be obtained from the transfer function of the structure. According to Ewins [27], the transfer function, may be defined as the contribution of the modal properties of rth mode, including mode shapes $\{\psi\}_r$, eigenvalues λ_r, natural frequency ω_r and modal mass scaling factor M_{Ar} [27, 28]:

$$[H(\omega)] = \sum_{r=1}^{N} \left[\frac{\{\psi\}_r \{\psi\}_r^T}{M_{Ar}(j\omega - \lambda_r)} + \frac{\{\psi\}_r^* \{\psi\}_r^{*T}}{M_{Ar}^*(j\omega - \lambda_r)} \right] \tag{1}$$

In Eq. (1), j denotes the imaginary part and stars correspond to the complex conjugate pairs of the variables. Theoretically, the transfer function is defined by an infinite number of modes. However, the transfer function is convergent by using a sufficient number of modes (N). As shown in Eq. (2), at ω=0, modal flexibility matrix is obtained [27, 28]:

$$[f] = [H(\omega = 0)] = \sum_{r=1}^{n} \left[\frac{\{\psi\}_r \{\psi\}_r^T}{M_{Ar}(-\lambda_r)} + \frac{\{\psi\}_r^* \{\psi\}_r^{*T}}{M_{Ar}^*(-\lambda_r)} \right] \tag{2}$$

In this study, the real values of modal properties obtained from the bridge modal test are used. Therefore, only the real parts of the mode shapes are utilized in Eq. (2). It should be mentioned that the lower-order modes are generally more important than the higher-order modes in the global vibration response of the bridge structures. Thus, only a limited number of vibration modes and degrees of freedom are selected for the practical applications [29]. To avoid losing critical information, the resolution should be considered carefully.

Since the modal flexibility matrix can be estimated from the first few modes of vibration of the structure without any complicated computations, many efforts have been made to find the damage identification methods based on this matrix. In the study, the flexibility-based damage detection methods are considered from the classical to the new approaches have been suggested by flexibility-deflections. The classical flexibility-based method was proposed by Pandey and Biswas [6]. Pandey and Biswas [6] computed the maximum absolute value of flexibility changes in the pre- and post-damaged states. The flexibility change caused by the damage can be obtained from the difference in the respective matrices, i.e. [6]:

$$[\Delta f] = [f] - [f*] \tag{3}$$

In Eq. (3), $[\Delta f]$ represents the change in flexibility matrix, and the asterisk signifies the properties of the damaged structure. For each column of matrix $[\Delta f]$, the absolute maximum value of the elements in the jth column is defined by $\bar{\delta_j}$. Hence [6]:

$$\bar{\delta}_j = max[\Delta f_{ij}] \quad , \quad i=1,...,n \tag{4}$$

where Δf_{ij} are elements of matrix $[\Delta f]$ and are taken to measure the flexibility change at each measurement location. This classical method could not quantify damage and, in some cases, has false alarms for damage localization [6].

According to Eq. (5), the structural deflections $\{\delta\}$ can be obtained by multiplying the spatial load distribution vector $\{F\}$ by the flexibility matrix $[f]$. The deflection vectors extracted from the flexibility matrices are superior in the damage identification process using modal flexibilities. These vectors, which can be recognized as the first Ritz vectors, provide a better understanding of damage detection problems [10].

$$\{\delta\} = [f]\{F\} \tag{5}$$

3.Proposed method based on modal flexibility deflections

In this section, a new method utilizing flexibility deflections is presented for damage identification of beam-like structures. The flexibility deflections are extracted from modal inertia forces. These flexibility deflections are capable of detecting and locating damage precisely. For quantifying damage severities, the method is developed by the concepts of static deflections and changes in the strain energies in beams. This research develops a two-stage approach to locate and estimate damage severity in bridge girders.

3.1. Damage localization based on Flexibility Deflections due to the modal inertia forces

In the present study, a new load distribution vector is suggested to obtain the flexibility deflections of beam-like structures. In the proposed method, modal inertia forces are substituted for the spatial load distribution vector in Eq. (5). As seen in Fig. 1, for each vibration mode, the modal inertial forces $-f_{I_n}$ act on degrees of freedom. According to Eq. (6), the structure can be assumed as a pseudo-static system in which the right-hand side represents the modal inertia load vector $-f_{I_n}$ [30, 31]. The inertia load vector is obtained by eigenfrequencies ω_n, mass matrix M, and deflections $\hat{v}_n$. In the left-hand side of Eq. (6), the product of stiffness matrix K and deflections $\hat{v}_n$ results in elastic forces [32]. Therefore, the free-vibration motion may be considered to involve deflections $\hat{v}_n$ produced by inertial forces acting on degrees of freedom. By the proposed method, the effect of each vibration mode of the structure on the damage patterns can be introduced in the mathematical framework.

$$K\hat{v}_n = \omega_n^2 M\hat{v}_n = -f_{I_n} \tag{6}$$

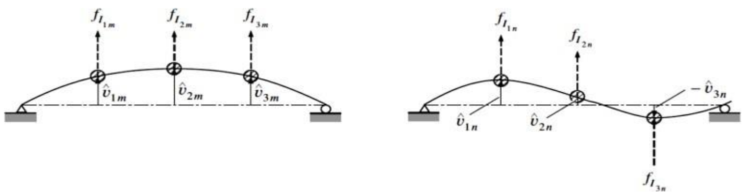


Fig 1. Vibration mode shapes and resulting inertial forces [32].

Using the modal flexibility matrix $[f]$, the deflection vectors $\{r\}_i$ corresponding to i-th vibration mode in pre- and post-damage damage states can be computed as [4, 29]:

$$\{r\}_i = [f]\{P\}_i \quad , \quad \{r\}_i^* = [f]^*\{P\}_i^* \tag{7}$$

In Eq. (7), $\{P\}_i$ is a spatial load distribution vector that involves modal inertia forces of mode *i*. The asterisks denote the post-damage state of the structure. The differences of the deflection vectors $\{r\}_i$ corresponding to the pre- and post-damage states form the damage identification vector $\{d\}_i$ of the structure:

$$\{d\}_i = \{r\}_i^* - \{r\}_i \tag{8}$$

The components of the damage identification vector $\{d\}_i$ indicate the candidate damage elements in the structure. The damage detection problem can be interpreted as a pattern recognition problem [4]. The $\{d\}_i$ variables are affected by uncertainties, and thus a statistical criterion should be applied for damage localization. In the present study, the average value of $\{d\}_i$ vectors for n modes of vibration is adopted for damage localization and severity estimations as the followings:

$$\{\bar{d}\} = \frac{1}{n}\sum_{i=1}^{n}\{d\}_i \tag{9}$$

A statistical criterion based on the evaluation of the following index zj is adopted in the process of damage identification. Using statistical inference approaches, the elements of $\{\bar{d}\}$ can be assumed as random variables with the mean of $\mu_{\bar{d}_j}$ and standard deviation of $\sigma_{\bar{d}_j}$. The standardized form of $\bar{d}_j$ variables is obtained by Eq. (10):

$$z_j = \frac{\bar{d}_j - \mu_{\bar{d}_j}}{\sigma_{\bar{d}_j}} \tag{10}$$

where zj is the standardized damage index for the j-th element. It should be mentioned that the zj variables introduce a damage pattern for the structure under consideration. The damage detection problem is interpreted as a pattern recognition problem [1, 4]. In pattern recognition problems, the normal distributions are the basis of the classification algorithms. The classification methods can be categorized in Bayes' rule, nearest distance, or hypothesis testing [1, 33]. A damage index is defined by a hypothesis test statistic. Hypothesis tests are advantageous for determining the distinction between the potential damage elements from undamaged in a bridge girder. The hypothesis testing provides an appropriate mathematical framework for deciding with a limited set of data [34]. The normalized damage indices zj are compared to $z\eta$, which represents the level of significance of the test. The parameter $z\eta$ is considered as a discriminating or threshold level that the decisions are made based on this criterion [33]. In addition, a hypothesis test enables the evaluation of the likelihood of making Type I and Type II errors. It should be mentioned that when the changes of natural frequencies are very slight, for convenience, the modal inertia forces can be normalized by the square of natural frequencies. This phenomenon is observed frequently in real badge structures.

3.2. Damage severity estimation based on changes of flexibility-delectations (CFD)

In the present study, the damaged elements in bridge girders are diagnosed by the changes in modal flexibility deflections and hypothesis tests. For evaluating the damage severity, a new method is proposed utilizing the static deflection concepts of beam-like structures. Most of the damage identification methods based on the static responses use iterative optimization-based techniques with a very high computational cost [35-37]. Few methods have been developed without using complicated optimization procedures and finite element models [20, 38].

It should be mentioned that measuring static deflections in bridge girders is a challenging task, especially, in traditional methods of contacting sensors. However, the static deflections can be obtained approximately by modal flexibility matrices [20]. This is one of the advantages of flexibility deflections in structural health monitoring. In this research, the flexibility deflections due to the modal inertia forces are assumed as static deflections in the modal space. According to the concepts of structural analysis, static deflection is a fundamental parameter for reflecting the stiffness changes of the structure. As observed in Fig. 2-a and Eq. (11), the undamaged deflection of the beam (u^{ud}) at location x is determined by the principle of Virtual Work. In the Virtual Work method, $M(x_1)$ and $m(x_1)$ are bending moments due to the real and virtual unit point loadings [38].

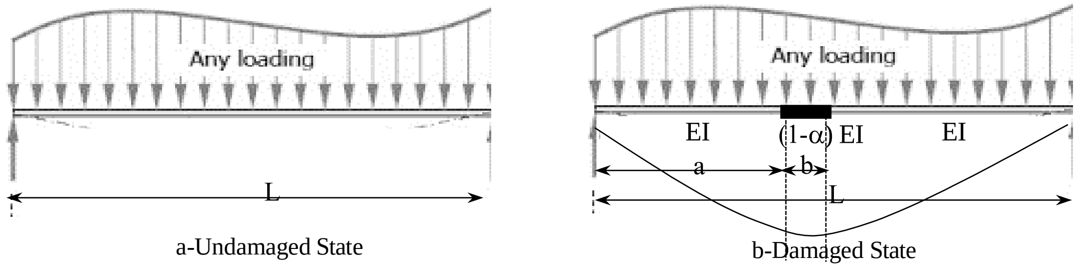


Fig 2. Beam deflections due to the vertical loading in pre- and post-damage states

$$u^{ud}(x) = \int_0^L \frac{M(x_1)\cdot m(x_1)}{(EI)_x}dx_1 = \int_0^L \frac{M(x_1)\cdot m(x_1)}{EI}dx_1 \tag{11}$$

The damage state in a beam is introduced by damage severity coefficient 0≤α<1 for bending rigidity in a segment $a \le x < a+b$ (Fig. 2-b). Thus, from the Virtual Work theory, the beam deflections at damage state ($u^d(x)$) is obtained by Eq, (12) and Eq, (13) as below [38]:

$$u^d(x) = \int_0^a \frac{M(x_1)\cdot m(x_1)}{EI}dx_1 + \int_a^{a+b} \frac{M(x_1)\cdot m(x_1)}{(1-\alpha)EI}dx_1 + \int_{a+b}^L \frac{M(x_1)\cdot m(x_1)}{EI}dx_1 \tag{12}$$

From Eq. (13), the elastic deflection changes (*DC*) are obtained along the beam structure.

$$u^d(x) = u^{ud}(x) + \int_a^{a+b}\left(\frac{1}{1-\alpha}-1\right)\frac{M(x_1)\cdot m(x_1)}{EI}dx_1 \tag{13}$$

It is evident that *DC(x)* pattern is to be able to determine the damage location and severity estimation. As shown in Eq. (14), the *DC(x)* function is formulated by the Virtual Work theorem and damage severity parameters α and β [38].

$$DC(x) = u^d(x) - u^{ud}(x) = \beta\int_a^{a+b} \frac{M(x_1)\cdot m(x_1)}{EI}dx_1 \quad , \quad \beta = \frac{\alpha}{1-\alpha} \tag{14}$$

Application of Eq. (14) for continuous beams with arbitrary loading is complicated and time-consuming. Here, the *DC(x)* pattern produced by modal flexibility deflections due to the inertia forces is utilized for determining the damage locations in the bridge girders. The pattern that is produced by flexibility deflection changes, is named *FDC(x)*. The total area enclosed by *FDC(x)* can show the progressive damage. In fact, the integral of the difference between the beam deflections of the damaged u^d and healthy u^{ud} states, Δ, is the total response of the beam due to the lack of stiffness in the damaged segment:

$$FDC(x) = u^d_{FD} - u^{ud}_{FD} \quad , \quad \Delta = \int_0^L FDC(x)dx \tag{15}$$

Besides the changes in deflections, the changes in strain energies before and after the damage is a useful tool to determine the damage severity. In this paper, a severity estimation method is proposed by the strain energies of the flexibility deflections due to the modal inertia forces. According to Fig. 2, the total strain energies before damage U^{ud} and after damage U^d are determined by Eq. (16) and Eq. (17).

$$U^{ud} = \int_0^L \frac{M^2}{2EI}dx \tag{16}$$

$$U^d = \int_0^L \frac{M^2(x)}{2EI} dx + \int_a^{a+b} \left(\frac{1}{1-\alpha} - 1\right) \frac{M^2(x)}{2EI} dx \tag{17}$$

The change of strain energy of flexibility deflections (UC_{FD}) can be expressed by Eq. (18). Where β is the damage severity index.

$$UC_{FD} = U_{FD}^d(x) - U_{FD}^{ud}(x) = \sum_{j=1}^m \int_{a_j}^{a_{j+1}} \beta \frac{M^2(x)}{2EI} dx = \beta \sum_{j=1}^m U_{FD\,j}^{ud} \tag{18}$$

It is worth that in beams with the multiple damaged areas, the positive changes in strain energies are accounted for severity estimation. In this paper, the strain energy of the beam structure is obtained by the curvatures of the modal flexibility deflections. The vector of flexibility, deflection r_i and the vector of curvatures of bending strains κi are obtained by Eq. (19). The strain energies U_i and U_{ij} are calculated from Eq. (20) for the beam and segment j [33].

$$\kappa_i = \frac{\partial^2 r_i}{\partial^2 x} \quad , \quad \kappa_i = \frac{M}{EI_i} \tag{19}$$

$$U_i = \frac{1}{2}\int_0^L [EI\kappa_i{}^2]dx \quad , \quad U_{ij} = \frac{1}{2}\int_{a_j}^{a_{j+1}} [EI\kappa_i{}^2]dx \tag{20}$$

3.3. The novel two-stage damage identification method

The proposed damage detection method is a non-model-based damage identification approach that does not rely on dynamic updating procedures. The theoretical basis of the proposed method is clear and suitable for practical applications. All of the steps of the proposed method for damage identification are summarized in Fig. 3. It should be mentioned that the new spatial load distribution vector in stage 1, is independent of the uniform load surface method. For severity estimations in stage 2, the very useful damage parameters α and β are utilized. Formerly, these parameters were applied to static deflection shapes of Euler-Bernoulli beams [38]. By computing the change of strain energy of flexibility deflections (UC_{FD}) in Eq. (18), the values of damage parameters α and β are determined. These new modifications were not utilized in the previous studies [2, 39].

The feasibility of the novel damage identification method is demonstrated by the modal data of the two full-scale bridge girders. First, the flexibility deflections of the steel north plate girder of I-40 Bridge over the Rio Grande river are considered and discussed. Second, the damaged surface areas of a pre-stressed concrete box girder that was removed from the Provincial Highway Bridge over the Qu'Appelle river are identified by the proposed method.

4. Field study on north plate girder of I-40 bridge

The performance of the proposed flexibility-deflection-based method is evaluated by using the field data from the simulated damage on I-40 Bridge over the Rio Grande river [3]. According to the elevation view of the I-40 Bridge in Fig. 4-a, the bridge consisted of twin spans for two bounds of traffic. The spans of each bridge were comprised of a concrete deck supported by two welded, steel, plate-girders, and three stringers (Fig. 4-b). As shown in Fig. 4-b, the loads were transferred by the stringers from the deck to the plate girders by floor beams located at 6.096 m (20 ft) intervals. Each bridge was constructed of three structurally independent parts [3, 40]. A single part had three spans consisting of the two end spans of 39.624 m and the middle span of 48.768 m (Fig. 5) [3]. Both north and south bounds of I-40

Bridge, were demolished in 1993 and replaced by a new bridge (Farrar and Jauregui, 1996). Before demolishing, during the summer of 1993, New Mexico State University and Los Alamos National Laboratory conducted an extensive experimental program to simulate damage in the main plate girder of I-40 Bridge and explore various damage identification methods [3].

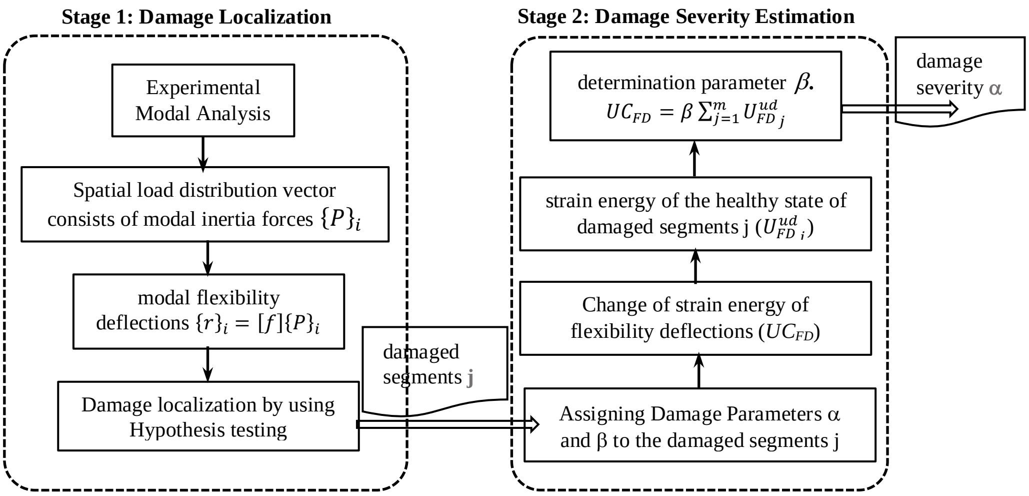
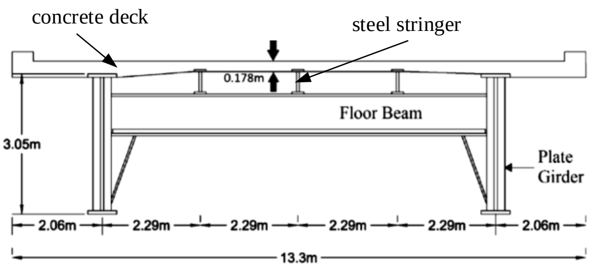


Fig 3. The proposed two-stage damage identification algorithm

a-I-40 Bridges over the Rio Grande in Albuquerque, New



b- Cross-section geometry of I-40

Fig 4. Elevation view Cross-section geometry of I-40 Bridge [3]

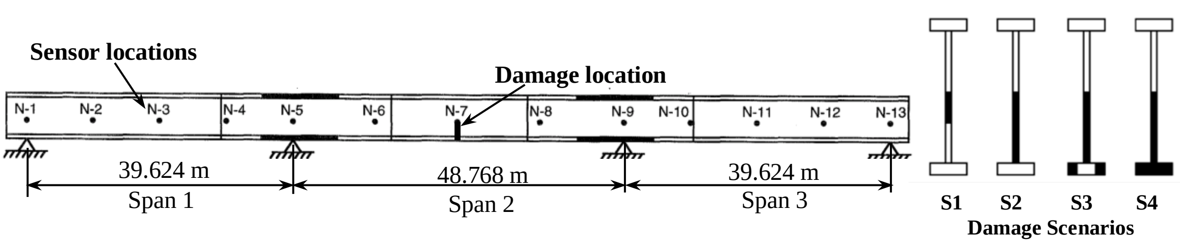


Fig 5. Sensor locations and damage scenarios in the north plate girder of I-40 Bridge [3].

In the vibration tests the sequential damages were introduced to the bridge, and the fatigue crack growth was simulated in the main plate girder (Fig. 5). Progressive damage was simulated by making various torch cuts in the web and flange of the middle span of the north plate girder (Table 1) [3].

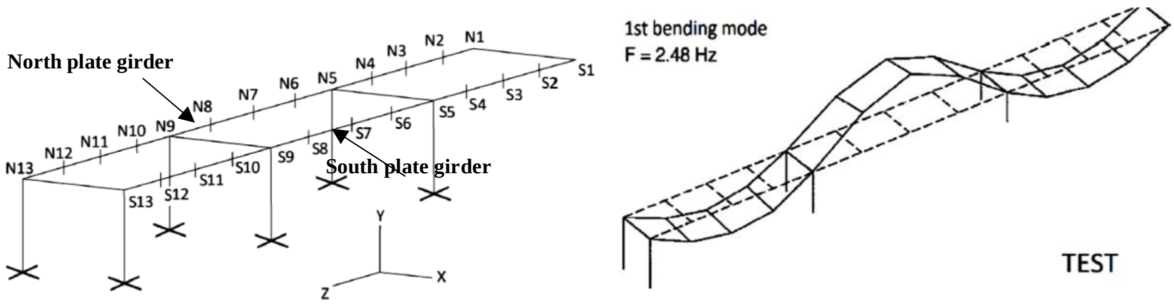


Fig 6. Locations of the accelerometers and the first vibration mode of I-40 bridge [3, 40].

Table 1- Eigenfrequencies of the north girder of I-40 Bridge through damage scenarios [3].

Scenario	Damage description	Frequencies (Hz)					
		Mode1	Mode2	Mode3	Mode4	Mode5	Mode 6
0	Undamaged	2.48	2.96	3.50	3.08	4.17	4.63
1	61 cm cut at the center of the web	2.52	3.0	3.57	4.12	4.21	4.69
2	182 cm cut in the web to the bottom flange	2.52	2.99	3.52	4.09	4.19	4.66
3	182 cm cut of the web and half of bottom flange cut	2.46	2.95	3.48	4.04	4.14	4.58
4	182 cm cut of the web and entire bottom flange cut	2.30	2.84	3.49	3.99	4.15	4.52

Before any simulated damage, forced vibration tests were carried out on the bridge structure. Subsequently, forced vibration tests were repeated for each progressive damage scenario. The input force was provided by a 9.7861 kN hydraulic actuator, and a random-signal generator was used to produce 8.8964 kN peak-force uniform random signal over the frequency range 2–12 Hz [3]. The output acceleration time histories in the vertical direction were measured by a coarse mesh of accelerometers. A limited number of sensors were used for data acquisition system. As observed in Fig. 5 and Fig. 6, the accelerometers were placed at mid-height of the plate girder web with a nominal spacing of 9.906 m (32.5 ft) in the side spans and 12.192 m (40 ft) in the mid-span [3]. Actually, the vibration responses of the middle span are monitored only by three sensors. Mode shapes were extracted from the cross-spectral of the various accelerometer readings. The measured fundamental vibration mode is shown in Fig. 6. Damage is associated with a decrease in the eigenfrequencies, increases in the damping values, and alternations of the modes of vibration of the structure. According to Table 1, no significant changes occurred in the eigenfrequencies of the north plate girder. The eigenfrequencies have shown little promise for detecting the presence of damage.

In this paper, data collected from the mentioned first through fourth levels of damage is used to evaluate the field applicability of the damage identification methods. Since modal data for the undamaged condition of the bridge are available, no attempt is made to build a numerical model of a baseline structure.

4.1. Damage localization in north plate girder of I-40 bridge

In this section, the accuracy of the proposed flexibility-based damage identification method is evaluated using experimental data from the 1-40 Bridge during sequential damage scenarios. Furthermore, the uniform load surface method and the classical Pandey and

Biswas method examined and compared to the proposed method. Comparative studies demonstrate the advantages of the proposed method with respect to uniform load surface and the classical Pandey and Biswas method.

The influence of the number of vibration mode shapes on flexibility-based damage indices is studied and discussed. It is found that the first two vibration modes are very influential on the damage pattern, and incorporating data from higher modes was not found to improve the localization performance. The modal flexibility matrices of the north plate girder were calculated based on the first two vibration modes during sequential damage scenarios. As shown in Fig. 7, changes in the flexibility matrices during the first three damage scenarios are very slight. Significant changes are observed in the components of the flexibility matrix in the fourth damage scenario (Fig. 7). By utilizing the flexibility-based damage indices, these changes reveal the location and severity of the damage.

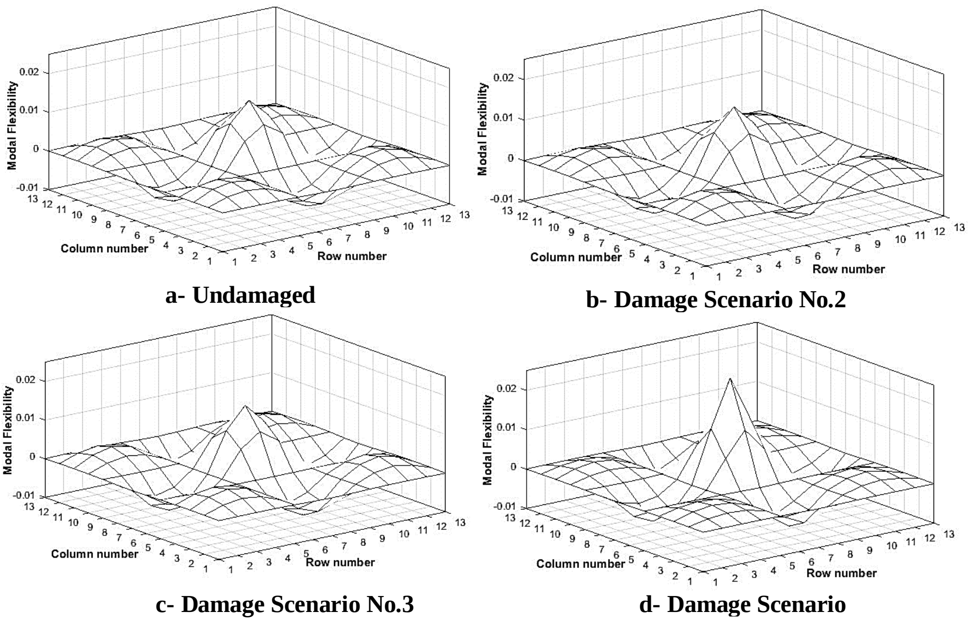


Fig 7. The effects of progressive damage on the components of modal flexibility matrices

In the following, three flexibility-based damage patterns are selected to detect the candidate damage elements in the north plate girder of the I-40 Bridge. These patterns are obtained by the method presented in this_study, uniform load surface method, and the classical method that was proposed by Pandey and Biswas. The components of the damage patterns are indices that show the damaged elements of the structures. The damage indices are assumed as random variables with a specified mean and standard deviation. To study the performance of damage indices and determine the damaged elements, these variables are standardized by Eq. (10). Two criteria corresponding to the one-tailed statistical test are applied for the analysis. Two values of 1.5 and 2 are specified for z scores (z_η). The value of 1.5 corresponds to a significance level of 0.0668 or 93.32% confidence level. The score of z_η=2, reflects a higher level of confidence 97.7% and low errors (significance level of 0.023). The values of z_η determine the damage threshold level. In addition, the localization resolutions of the methods are examined and discussed by these criteria.

Using the experimental modal data of the four damage scenarios of the I-40 Bridge, the flexibility matrices and modal inertia forces are calculated through Eq. (7), and the flexibility deflections are obtained. The normalized flexibility index for each degree of freedom is obtained by Eq. (9) and Eq. (10). This new damage detection technique shows to be able to locate damage in the north plate girder. Fig. 8 shows that by choosing the threshold level of damage z_η=2, the flexibility deflection index reveals the early-stage damage on the steel plate girder, and the exact location of the damage is obvious. The flexibility deflection-based damage indices reach a maximum in the vicinity of the damage region (node 7). At the threshold level of z_η=1.5, there are a few false alarms along the girder.

The problem of selecting a suitable number of modal data for the proposed method to cause no false alarm is considered and discussed. Next, the uncertainty in the mode shapes as input to the damage-detection method has a relatively strong influence on the damage-localization error. In most situations, it was found that the contributions of particular modes to the overall vibration behavior of a structure are different. As discussed in Section 2**,** the flexibility matrix is more strongly influenced by the lower frequency modes. In this field data analysis, an accurate representation of flexibility-based indices can be obtained with first two measured modes. As observed in Fig. 8, there are virtually no improvements in the damage indices when additional modes are included. By using the first two modes of vibration, the very promising results are obtained by the present study with a higher level of confidence 97.7% (z_η=2).

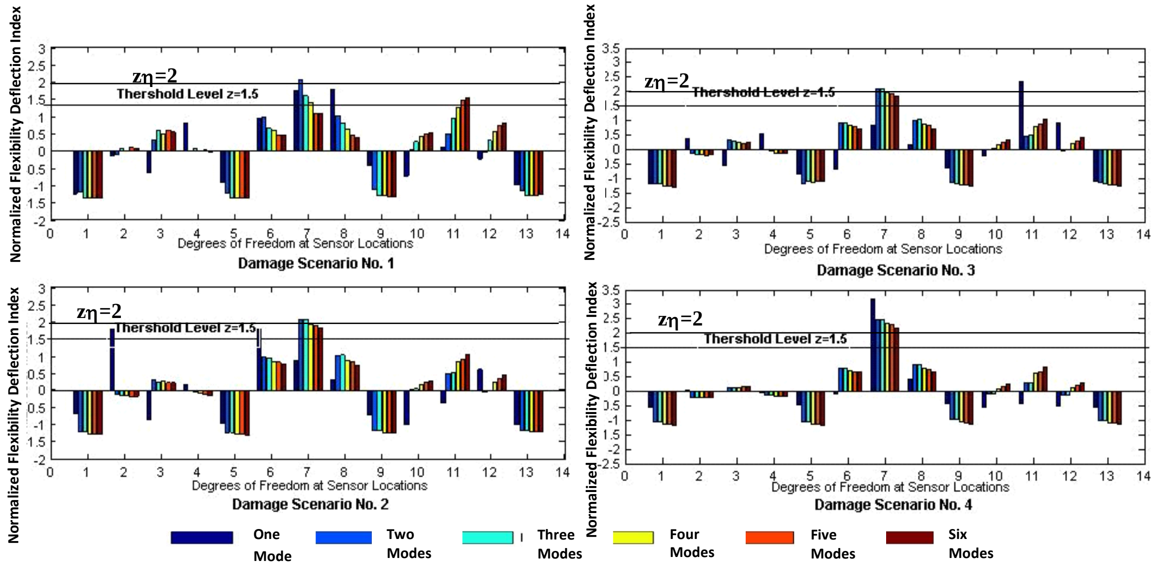


Fig 8. The present study based on flexibility deflections for the north plate girder of I-40 Bridge during sequential damage scenarios

For comparative study, one of the flexibility-based methods that have many potential advantages in structural dynamics is utilized and discussed here. The uniform load surface method is applied to the experimental modal data of 1-40 Bridge. A uniform unit vector is applied for the spatial load distribution vector in Eq. (7). The flexibility deflections extracted from the unit load vector are used to obtain the normalized damage index. The results are shown in Fig. 9. The results indicate that the localization resolution declines compared to the results of the proposed flexibility deflection-based method (Fig. 8). In the first three levels of damage scenarios, the damaged area is not revealed by the uniform load surface at a level of confidence 97.7% (z_η=2). At the threshold level of z_η=1.5, the damaged area during the first three scenarios is revealed with some false alarms along the girder. Comparison of the bar plots in Fig. 9 and Fig. 8 demonstrates the advantages of the present study. In uniform load surface analysis, the more accurate results are also obtained with first two measured vibration modes.

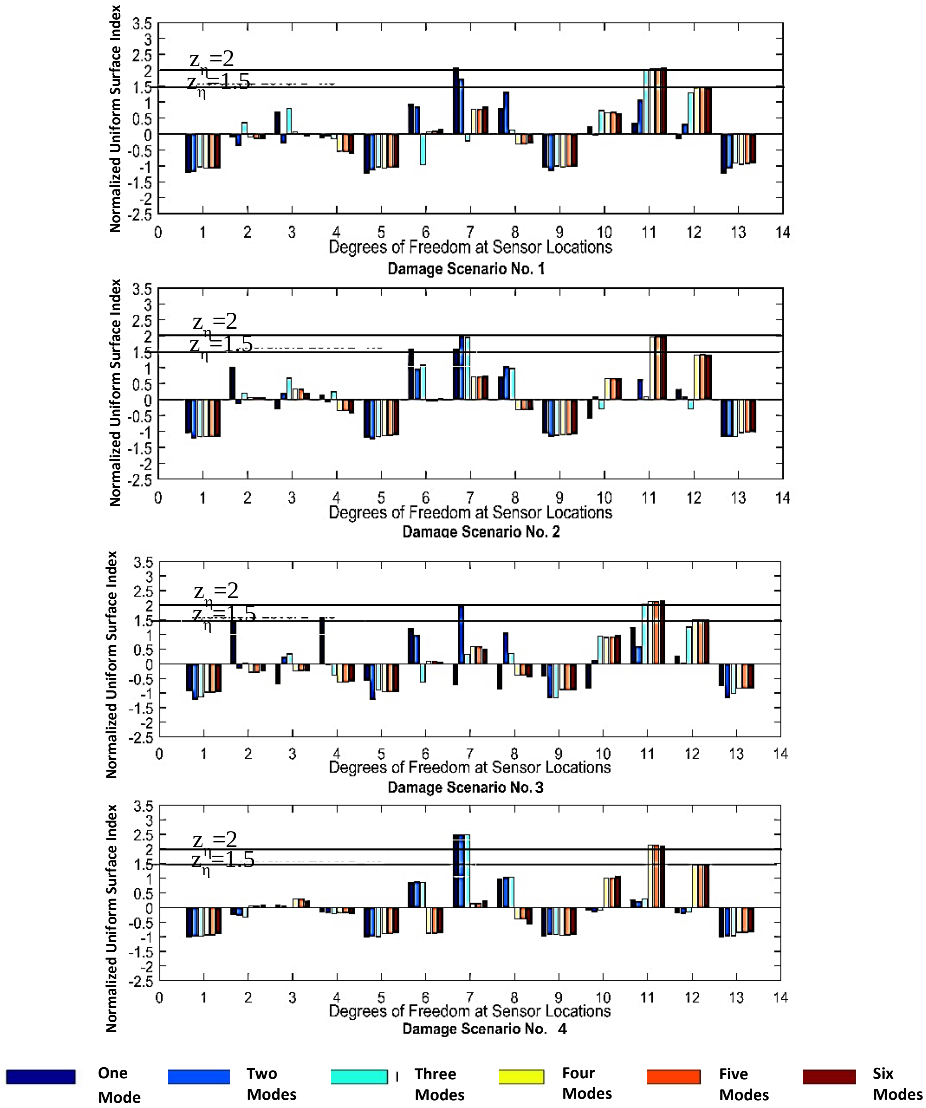


Fig 9. Normalized uniform load surface damage indices for the north plate girder of I-40 Bridge during sequential damage scenarios

The abilities of the classical flexibility-based damage index proposed by Pandey and Biswas [6] are investigated and discussed here by experimental modal data of I-40 Bridge. The Pandey and Biswas [6] damage indices are calculated by Eq. (3) and Eq. (4) in Sec.2. The normalized indices are presented in Fig. 10. As observed in Fig. 10, those indices show little promise for detecting the presence of damage.

Especially at the level of confidence 97.7% (z_η=2), the two early stages of damage are not detected. At these two damage scenarios, the damage index values are false alarms. Only in the final step, the damage location is predicted correctly. In step 3 of damage scenarios, the true damage estimation is obtained by two modes of vibration. Again, the resolution of Pandey and Biswas [6] method is better for the first two vibration modes of the I-40 Bridge.

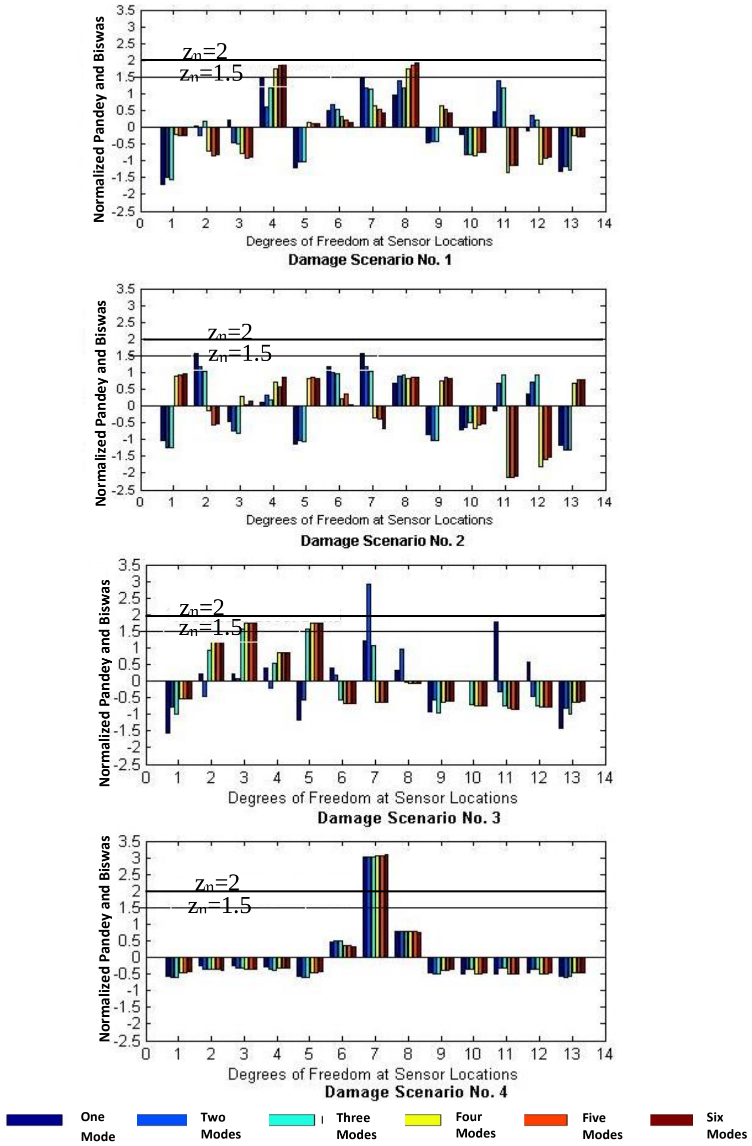


Fig 10. Normalized Pandey and Biswas damage indices for the north plate girder of I-40 Bridge during sequential damage scenarios

4.2. Damage severity estimation in north plate girder of I-40 bridge

In bridges with several spans, the changes of the flexibility deflections through progressive damage scenarios are rather complex. Thus, it is more appropriate to partition off the longitudinal axis of the bridge into its spans and investigate each domain separately. In other words, the structure is divided into substructures. For damage severity estimation in the north plate girder of the I-40 bridge, the side and middle spans are studied separately. The mode shapes were measured at thirteen sensor locations in the dynamic tests. Hence, in each span, a data set consisting of five measurement points is available. The coarse mesh of sensors is not adequate for the severity estimation of damage. Another important issue is the reduction of experimental noise in the modal data. Utilizing the raw data directly in the severity estimation analysis, may lead to inaccurate results. In this study, the cubic spline interpolation technique is applied to filter out this noise and obtaining smoothed mode shapes. The cubic spline functions can extrapolate additional measurement points [41, 42]. The cubic spline functions provide a good approximation of the beam deflections, mode shapes, and their derivatives [42]. Utilizing the spline functions, the measurement points of span 1 and span 3 were increased to

27, and the measurement points of span 2 were increased to 41 points. The flexibility deflections due to the modal inertia forces are obtained for each bridge span. Due to the very small changes in natural frequencies, shown in Table 1, the modal inertia forces are normalized by the square of natural frequencies. The plots in Fig. 11 show these flexibility deflections for the four levels of damage scenarios in the north plate girder of the I-40 Bridge. According to the plots in Fig. 11, a significant change in the flexibility deflections of span 2 is observed through progressive damage scenarios. There are no significant changes in the flexibility deflections of span 2 and span3 (Fig. 11).

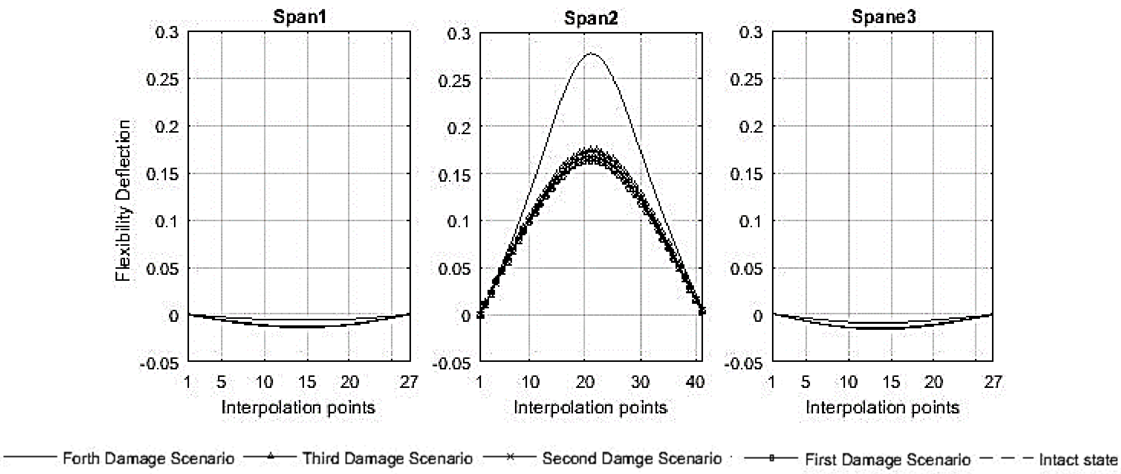
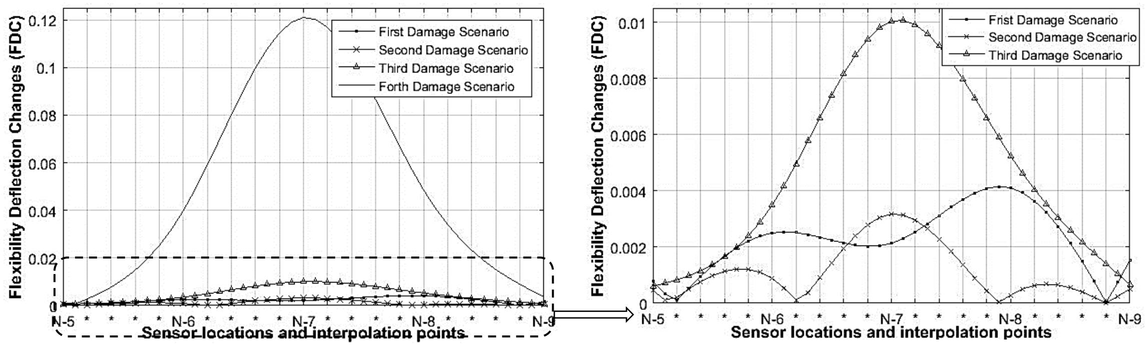


Fig 11. Flexibility deflections in three spans of the north plate girder of the I-40 Bridge due to the fundamental modal inertia forces

The *FDC(x)* damage patterns of span 2 in the north plate girder of the I-40 Bridge for the four damage scenarios are shown in Fig. 12. These patterns were produced by modal flexibility deflections due to the modal inertia forces. As observed in Fig. 12, the area enclosed by *FDC(x)* can show damage progress in the girder. The deflection changes in the first three damage scenarios are slightly compared to the final damage scenario.



a-Four damage scenarios **b- First three first damage**

Fig 12. Flexibility deflection changes (FDC(x)) in span 2 of the north plate girder of the I-40 Bridge

For damage severity estimation, the changes of strain energy of flexibility deflections (UC_{FD}) for the four levels of damage in the north plate girder are computed by Eq. (18). In the damaged region j of the plate girder, the strain energy of the healthy ($U^{ud}_{FD\,j}$) is also calculated by Eq. (18). From these quantities, the values of parameter β are obtained for each damage scenario. Subsequently, the values for damage severity estimation parameter α are determined. The results of damage severity estimations (α values in percent) are presented in Table 2. For comparative study, the severity estimation results are compared to the true values and those estimated by Dincal and Stubbs [43]. They determined the damage severities by the Invariant Stress method. According to Table 2, promising results are obtained by the method proposed in this paper. The results of predictions based on the newly proposed method are more accurate than those obtained by Dincal and Stubbs [43].

Table 2-Assessment of the severity estimations for damage scenarios in I40-North plate girder (percent reduction of bending rigidity)

Damage Scenario	Present study		Dincal and Stubbs [40] Invariant stress method		True damage severity %
	Predicted	Error %	Predicted	Error %	
S-1	1.58	1.38	35.2	35	0.2
S-2	28.2	16.3	24.9	13	11.9
S-3	36.62	7.38	29.7	14.3	44.0
S-4	93.26	3.04	88.00	8.3	96.3

5.Pre-stressed concrete bridge box girder

In construction of many bridges, pre-stressed concrete girders play an essential role in the load carrying-capacity. Hence, health monitoring of the full-scale pre-stressed concrete bridge girders is necessary to find a reliable and efficient damage detection method. In this section, the feasibility of the proposed method is examined by the results of dynamic tests that were carried out on a full-scale pre-stressed concrete girder in the Structures Laboratory at the University of Saskatchewan [44]. Saskatchewan Highways and Transportation Administration donated several pre-stressed concrete bridge girders removed from the Provincial Highway Bridge over the Qu'Appelle river to the Structures Laboratory (Fig.13-a) [45].

a- Bridge on Provincial Highway No. 56 in southern Saskatchewan prior to girder replacement

b- The dynamic test set-up for the removed prestressed concrete box

Fig 13. The Provincial Highway Bridge over the Qu'Appelle river and its removal pre-stressed concrete box girder [45]

The girder is 12.2 meters long, spanned 11.9 meters, and had a standard double box 1216×508 mm cross-section, as shown in Fig. 14 and Fig. 15-b. According to the design drawings for the girder, compressive strength of 34.5 MPa and Young's modulus of E=26.1 GPa were adopted for the concrete [44]. Vibration tests were carried out under well-controlled conditions in the Structural Laboratory in the College of Engineering at the University of Saskatchewan. The vibration responses of the simple support concrete girder were measured by fourteen accelerometers at equally spaced locations on both sides of the top of the specimen [44] (Fig. 14). The artificial damage was done by removing two small square blocks of concrete from the top surface. These small blocks with dimensions of 150 ×150 mm in plan and 35 mm deep, introduced a very low level of damage, such as surface deterioration that occurs in bridges [44, 45]. This surface damage causes a local reduction in the flexural rigidity of approximately 2.49 %. The fundamental vibration frequencies and mode shapes of the pre-stressed concrete girder in pre- and post-damage states are presented in Fig. 15-a.

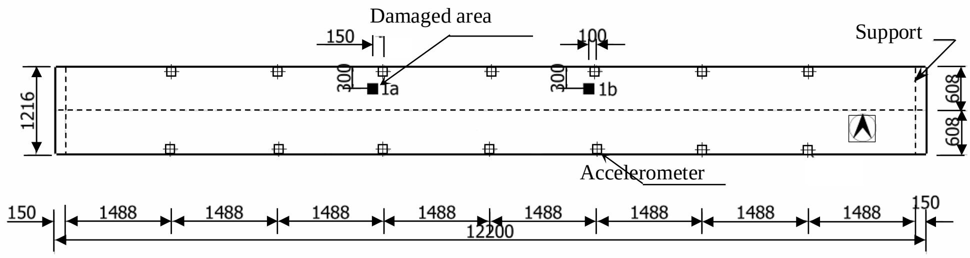
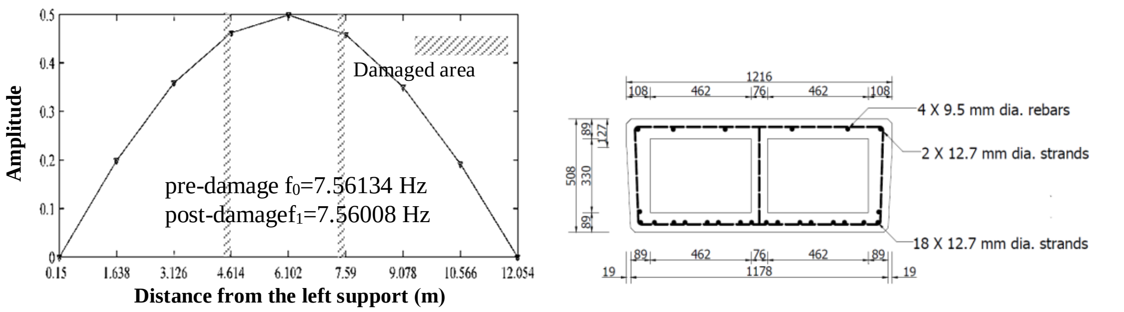


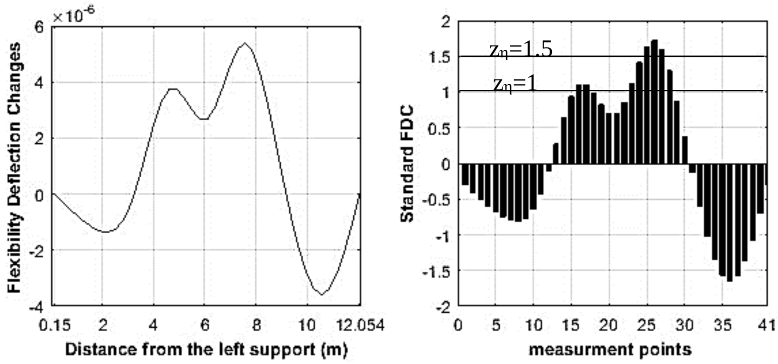
Fig 13. The plan of the pre-stressed concrete bridge girder in vibration test (dimensions in mm) [44, 45].



a- Fundamental mode shape and natural frequencies pre- and post-damage) **b- Cross-section of the girder (dimensions in mm)**

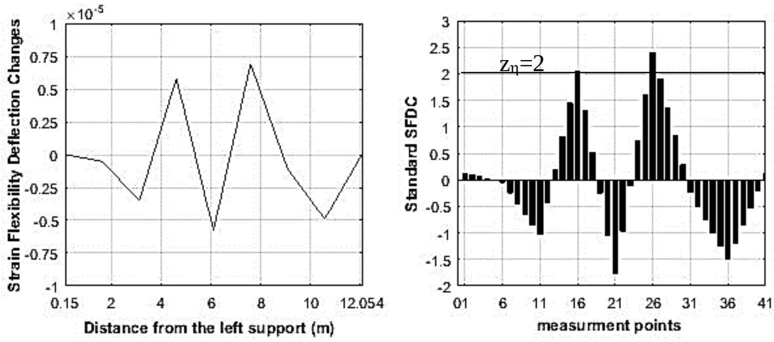
Fig15. Fundamental vibration mode and cross-section of the pre-stressed concrete bridge girder [44]

The fundamental vibration characteristics of the pre-stressed concrete girder are analyzed by the proposed flexibility-deflection-based method. Due to the coarse mesh of the measurement points, the measurement points were first increased to 41 points by spline interpolation functions. The deflection vector extracted from the flexibility matrix and modal inertia force vector. The diagram of flexibility deflection changes or *FDC(x)* along the pre-stressed concrete girder is depicted in Fig. 16-a. The damage pattern obtained by *FDC(x)* reveals significant changes near the two damaged surface areas on the top of the concrete girder. Using the Hypothesis test described in section 2 of this paper, the damage index of the girder is determined (Fig. 16-b). As shown in Fig. 16-b, the predicted damaged areas are near to the actual damaged areas of the girder. To increase the resolution of the proposed damage localization, the strains of flexibility deflections are calculated by the spline method. According to Fig. 17-a, the differences between the strains of flexibility deflections in pre- and post-damage states represent precisely the damaged areas. The normalized deflection strains, correspond well with two damaged areas of the concrete girder with a confidence level of 97.7% (Fig. 17-b).



a- flexibility deflection changes or **b- Normalized *FDC(x)***

Fig 16. Changes in the flexibility-deflections due to the inertia forces of the fundamental (first) vibration mode of the pre-stressed concrete bridge girder



a- Changes in strains of flexibility deflection or **b- Normalized**

Fig 17. Changes in the strains of flexibility-deflections due to the inertia forces of the fundamental (first) vibration mode of the pre-stressed concrete bridge girder

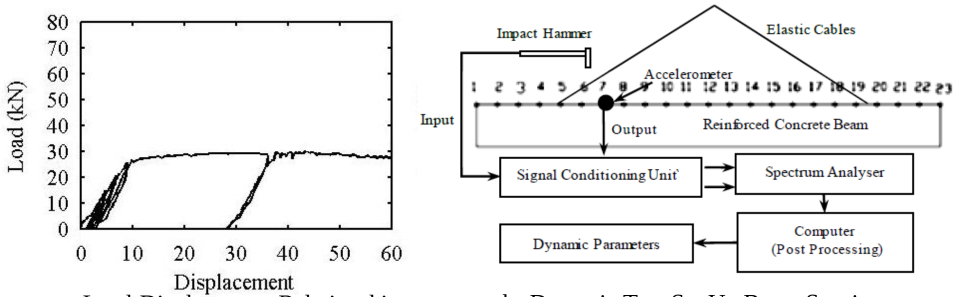
Estimation of the damage severity and the changes in strain energy of flexibility deflections (UC_{FD}) due to the surface damages are computed by Eq. (18). It should be mentioned that the positive changes of strain energies should be utilized for damage severity estimation. For the two identified damaged regions of the concrete girder, the strain energies of the healthy state ($U^{ud}_{FD\,j}$) are calculated using Eq. (18). Subsequently, the values of damage parameter β and the severity estimation parameter α are determined. The results of damage severity estimation (α values in percent) for pre-stressed concrete box girder are presented in Table 3.

Table 3-Assessment of the severity estimations for damage scenarios in pre-stressed concrete girder (percent reduction of bending rigidity)

Damage Scenario	Present study		True damage severity %
	Predicted	Error %	
S-1	0.98	1.51	2.49

6. Performance of the proposed method in the presence of modelling errors and noisy data

The feasibility and performance of the proposed method in the presence of modelling errors and noisy data is demonstrated using experimental and numerical data of a reinforced concrete beam. A reinforced concrete beam with dimensions of 150 mm wide, 200 mm high and 2200 mm long was manufactured and tested in dynamic structural laboratory. The flexural reinforcement consisted of two ϕ12 (12 mm diameter) steel bars at the bottom and top of the beam. Stirrups were made of ϕ8 (8 mm diameter) steel bars with center-to-center spacing of 100 mm.



a-Load-Displacement Relationship b- Dynamic Test Set Up Beam Specimens

Fig 18. Dynamic and static tests for the reinforced concrete beam

The concrete beam was gradually damaged through a four-point static test set-up. The static load incremented by 5 kN. After each loading step, the dynamic tests were carried out in suspended cases. As shown in Fig. 18-b, the dynamic test set-up comprised of a 2-channel analyzer, an impact hammer, a piezoelectric sensor and a personal computer. The degrees of freedom of the concrete beam were positioned on the longitudinal center line of the top surface. By using Frequency Domain Direct Parameter Identification (FDPI) method, the eigenfrequencies and mode shapes were determined. Natural frequencies of the reinforced concrete beam in the initial state L0 and load steps L1(First load step) and L4 (Before failure) are listed in Table 4.

Table 4. Natural frequencies of the reinforced concrete beam in initial state and load steps L1 and L4

Vibration Modes	Frequencies (Hz)		
	L0 (0 kN)	L1 (10 kN)	L4 (25 kN)
Mode 1	114.48	101.54	93.34
Mode 2	304.89	274.66	256.5
Mode 3	563.19	517.3	480.82

A two dimensional finite elements model was constructed to simulate the damage in reinforced concrete beam specimen. The simulation was carried out by ANSYS software. The plane elements (PLAIN 42) were used for concrete and the reinforcements were modeled by truss elements. The modelling errors are considered as 5% underestimation in the density of the concrete 15% underestimation in the modulus of elasticity of the concrete and 2% overestimation in the dimensions of the beam in the finite element model.

Table 5- Mechanical Properties of Materials

	Modulus of Elasticity (GPa)	Specific Weight (kg/m^3)	Poisson Ratio	f'_c (MPa)	f_y (MPa)
Concrete Properties	15	2400	0.17	20	----
Steel Properties	200	7850	0.3	----	400

The free vibration analysis was performed to obtain the modal characteristics of the beam. The numerical and experimental mode shapes in pre-damage state of the specimen are shown in Fig. 19 The frequencies of the beam specimen in pre-damage state from both experimental tests and simulation analyses are listed in Table 6.

Table 6- Natural frequencies of numerical and experimental reinforced concrete beam

Mode number	Natural frequencies (Hz)	
	Finite Element Analysis	Experimental results
Mode 1	115.05	114.48
Mode 2	302.29	304.89
Mode 3	562.65	563.19

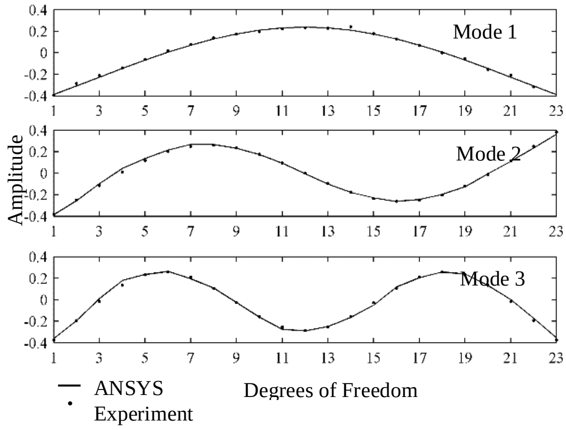
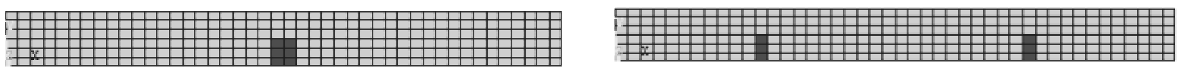


Fig. 19. Experimental and Numerical Mode Shapes of Reinforced Concrete Beam

Damage in the beam is numerically simulated by four different damage scenarios. In each scenario, the location and severity of damage were introduced by reduction factor Di for the elastic modulus of concrete in some of the elements. These damage scenarios are summarized in Fig. 20. The results of the proposed damage identification method for damage scenarios are shown in Fig. 21. It is observed that the damaged areas are detected well by the proposed method.



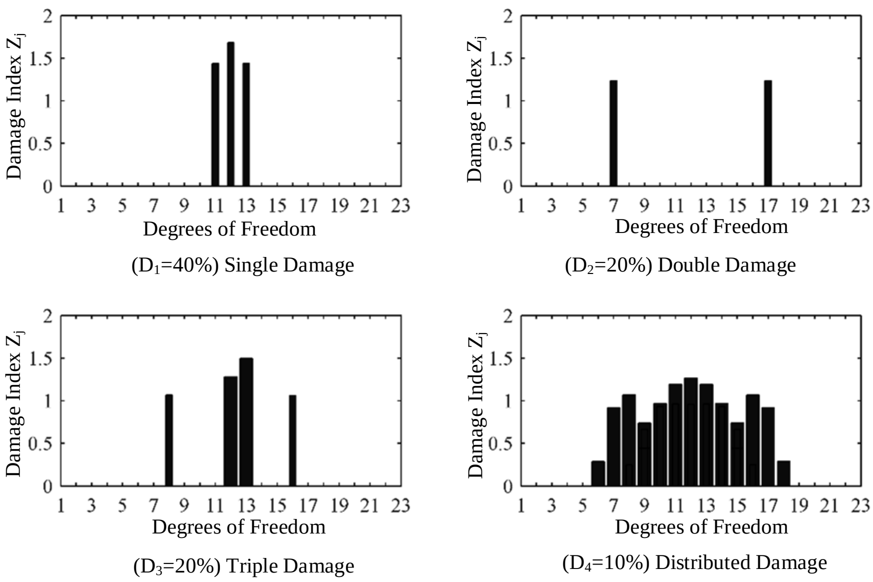
(a) Scenario1- D_1=40%

(b) Scenario 2- D_2=20%

(c) Scenario 3—D_3=20%

(d) Scenario 4- D_4=10%

Fig. 20. Damage locations and severity for Reinforced Concrete Beam



(D_1=40%) Single Damage

(D_2=20%) Double Damage

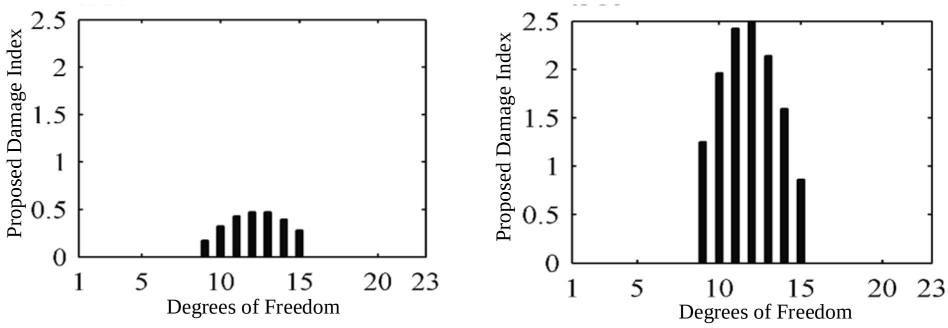
(D_3=20%) Triple Damage

(D_4=10%) Distributed Damage

Fig. 21. The results of the proposed damage identification method for the concrete beam

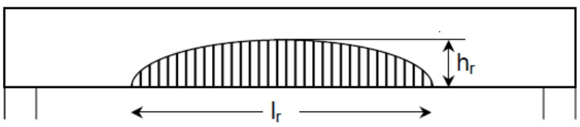
The noisy modal data of the reinforced concrete beam in the first loading step and before failure, are analyzed by the proposed method. Utilizing the experimental noisy modal data in calculating the flexibility matrices, yields inaccurate results. Therefore, the mixed approach is used here to remove the noise in experimental data. The mixed approach is a constrained optimization method that was proposed by Maeck [30]. In this method by using penalty factors in a constraint optimization problem, the noises are filtered out and the smoothed mode shapes are obtained accurately.

The flexibility deflection obtained by the modal inertia forces, can show damaged area, including the main cracks in the middle part of the concrete beam (Fig.22). It is interesting that the increasing size of middle cracks during load steps are reflected in damage patterns obtained by the proposed flexibility deflections. As observed in Fig.22, in load step L4 the size of damage pattern increases significantly.

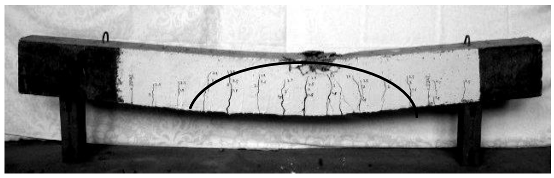


(a)- First load step L1 (b)- Forth Load Step L4

Fig. 22. Damage identification of concrete beam in the First load step and before failure



Crack pattern and Parabolic crack zone of a reinforced concrete beam proposed by Eilbracht [47]



Cracked area of reinforced concrete beam

Fig. 23. Crack pattern in reinforced concrete beam

The present study can detect the cracked area of the reinforced concrete beam in the presence of noisy data and the shape of the crack zone is in agreement with that suggested by Eilbarcht [46]. During the incremental loading, the bending cracks occur in the middle span of the reinforced concrete beam. Eilbarcht [46] proposed a parabolic-shaped crack zone for the crack patterns of the reinforced concrete beam (Fig. 23-a). It is evident that the length of the damaged area (l_r) does not change significantly during the progressive damage. However, the height h_r of the parabolic-shaped crack zone increases during the progressive damage. The decrease of the stiffness in the middle part of the concrete beam cause to create a zone of cracks. During the incremental load steps, the number of cracks does not significantly increase. In fact, the sizes of the cracks increase and lead to the failure. These assumptions, are consistent with the cracking process of the concrete beam in Fig. 23-a and the results of flexibility based damage identification in Fig. 22.

7. Conclusions

This paper focused on the application of flexibility deflections for damaged identification of beam- like structures. After a comprehensive review of flexibility based methods, a new method of applying the modal inertia forces to load distribution vector was presented. Based on the proposed load vector, a new two-stage damage detection method for beam-like structures was proposed. In the first stage the damaged elements are localized precisely and in the second stage the damage severities are estimated by introducing the damage parameters for the damaged area and computing the changes in strain energy.

To demonstrate the abilities and advantages of the proposed method, the experimental modal data of the steel plate girder of I-40 Bridge over the Rio Grande and of the pre-stressed concrete girder from the Provincial Highway Bridge over the Qu'Appelle river were selected and employed in two different case studies.

- Damage diagnosis of the north plate girder of I-40 Bridge during sequential damage scenarios indicate that the employment of modal inertia force vectors provides a good indication of the actual damage locations at the mid-span of the steel plate; from the early- stage to the final damage state. This is because the proposed method has a found theoretical basis, and the use of modal inertia forces as spatial load distribution vector has a better physical interpretation versus the uniform load surface method. The comparative studies demonstrate that the localization resolution declines between 15%-30% when the uniform load surface method is used.
- The results show that the localization resolution declines by the unit uniform load surface and the classical flexibility-based method. Especially, in the early stages of damage, the other two methods have some false alarms. The predictions of the present study are found to closely match the actual damage location with a high level of confidence of 97.7%. The first two vibration modes are very influential on the damage pattern and incorporating data from higher modes found not to improve the localization performance.
- Encouraging results were obtained by the curvatures of the proposed flexibility defections in identifying surface damage in a pre-stressed concrete box girder. The two damaged areas were localized precisely and quantified with 1.5% error by the new two stage damage identification method.
- Implementation of the concepts of static beam deflections in the new damage identification method based on the flexibility defections examined and discussed. Application of these concepts to the beam deflections led to the severity estimations with reasonable accuracy. The average of prediction errors in damage severity estimation is about 7%.
- The performance of the proposed method in the presence of modelling errors and noisy data was investigated. The experimental and numerical data of a reinforced concrete beam that was gradually damaged were analyzed. The results were consistent with the cracking pattern of the beam. The increasing size of the cracks in the middle part of the concrete beam reflected in the predicted damaged zone.

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