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Finite element model updating of bolted lap joints implementing identification of joint affected region parameters

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ABSTRACT

In this research, the new concept of ‘bolted joint affected region (BJAR)’ is introduced to simulate dynamical behavior of bolted lap joints. Such regions are modeled via special elements called contact zone element (CZE) which unify the neighboring contact surfaces of substructures. These elements are different from the thin layer interface elements that form an individual layer between the two substructures. The CZEs have no specified elastic characteristics. They are thus different from the adjoining solid elements and the constitutive relation for them is prescribed in normal and shear components. The unknown parameters of the model can be identified throughout model updating with modal test data. The structure’s frequency response function (FRF) is measured by excitation with an impact hammer and the measured responses are compared with model predictions including the CZEs’ parameters. The difference between the measured and predicted frequencies is minimized as the objective function. The optimized thickness and density are considered in addition to the elastic properties of BJAR. The competency of the proposed procedure is verified with modeling an actual structure containing a single lap bolted joint coupling two identical structural steel beams. The results showed proper conformity with model predictions. This model can be incorporated into the commercial finite element codes to simulate bolted joints for large and complex structures considering its accuracy and computationally efficient manner.

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Nomenclature

E	Young’s modulus of elasticity (N/m ²)	ν	Poisson’s ratio
G	Shear modulus (N/m ²)	λ	Eigenvalue ((rad/s) ²)
t	Thickness (mm)	ϕ	Eigenvector
W	Weight factor	ω	Natural frequency (rad/s)

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C	CZE stiffness matrix	σ	Normal stress (N/m ²)
K	Structural stiffness matrix	τ	Shear stress (N/m ²)
M	Structural mass matrix	ε	Normal strain
B	Strain-displacement matrix	γ	Engineering shear strain
ρ	Density (kg/m ³)		

1. Introduction

In industrial structures, the need for predictive models including jointed interfaces is crucial since such systems are composed of several thousand parts whose responses are determined by the joint and interface mechanics. The appropriate realization about the abstruse physics of bolted joint (BJ) surfaces as well as their modeling behavior and response to dynamical loading has been a serious concern for researchers during a couple of decades. BJs are one of the ordinary techniques in the assembly of engineering structures which affect their vibrational response and dynamical performance. Although their impact is to increase damping and to decrease the local stiffness of the structure and thus attenuating natural frequency resonances [1], their behavior is not yet known properly under dynamical loading condition. Precise modeling of BJs, as an essential factor for an accurate dynamical model, would not be easily accessible due to restrictions about understanding the mechanics of surfaces [2].

Despite the advances in computing power of processors and simulation capabilities of Finite Element (FE) softwares, there is still no particular element that can describe the exact BJ model in a dynamic case. The reason is that the physics of interfaces in BJs and the mechanism affecting their behavior are not adequately known yet. Few papers can be found in the literature about implementing FE softwares for BJ modeling. For instance, Kim et al. [3] have studied the precision of four BJ modeling techniques like node merging in interface and using beam elements in the classic ANSYS environment and have compared them to suggest various models in accordance with load cases.

Scientists have been always seeking appropriate models to simulate BJs because of uncertainties and their severe impact on dynamical behavior of structures [4]. Spring-damper [5-7], offset dimension [8], generic element [9], interface element (IE) [10], Jenkins element or Iwan model [11, 12] are all parametric approaches recently investigated by researchers. Bograd et al. [13] assessed the dynamics of mechanical joints through diverse modeling of Jenkins element, thin layer IE, etc. On a separate effort, Amir et al. [14] have compared modeling results obtained from analytical, statistical and FE techniques. BJ modeling based on their parametric or non-parametric attitudes is still a favorite theme for structural dynamics researchers. For example, Jalali [15] has recognized linear parameters of joining planes benefiting from the characteristic equation and has compared the results by Eigen-sensitivity method.

The interface layer concept has been emerged in regard to geology (e.g. rock slide) and drilling (soil-drill interaction) for the first time [16]. The concept comprises two main methods: thin layer IE and zero thickness IE. In the former, the joint behavior is to be controlled by considering a thin layer of unknown material between two sub-structures [17]. It should be noted that the behavior of such elements is assumed to be the same as the neighbors. Nevertheless, in the latter,

the interface layer thickness is taken as zero and then, a constitutive relation is defined usually between the shear and normal stiffnesses [18]. Today, this idea is rarely referred to, because of: inaccurate modeling of the joint behavior in normal direction [19], possibility of numerical instability [20] and some issues in meshing [21].

Using thin layer IE to identify mechanical joint surfaces in structural dynamics is proposed by Ahmadian et al. [22] for the first time which is currently a prominent method of joint modeling in FE model updating employing modal test data. For the interface layer, a fundamental assumption is that joining surfaces can be modeled through a thin film of an unknown material. Satisfactory results are frequently reported when applying the method to investigate the joints in structures. This technique has successfully been implemented in modeling of joints for large scale surfaces [23], BJs [24] and welded joints [25] so far. NASTRAN software is used for optimizing and minimizing the cost function throughout all these papers. The method is also adaptable to calculate joint damping experiments [26].

A serious concern in the IE method is the identification of parameters such as thickness and density. For the time being, there is no clear hypothesis to obtain the IE thickness; however, its optimum value depends on other factors like the joining surface roughness, roughness distribution, joint dimensions, preloading and finally the effect of neighboring elements [27]. Pande and Sharma [25] have shown that the ratio of the IE thickness to the average joint dimension can even reach 0.001 without prevailing a problem in calculations. However, Desai et al. [17] suggested the ratio to be within 0.01-0.1 range according to a parametric study. Jalali et al. [26] and Iranzad and Ahmadian [27] have taken the IE thickness equal to the constant value of distance between the neutral planes of the two bolted beams when trying to identify a 2D lap joint model. In [11], the IE density is taken a small number (say 1 kg/m^3) to avoid extra mass exertion to the structure.

To our knowledge, there cannot be found parameters such as the IE thickness and density as the updating parameters except elastic properties (e.g. the Young's and shear moduli) in the relevant researches. The main goal of this paper is to propose a simple model entitled Contact Zone Element to simulate bolted joint behavior under dynamical loading conditions together with investigation of their thickness and density effect on prediction of numerical models. Another aim of the article is to assess the proposed model performance in comparison with experimental records for a real structure utilizing the ANSYS software facilities.

2. FE modeling of contact area

In a BJ fastened by standard torque, there will not occur any sliding on the planes. However, it is subjected to low amplitude excitations and thus, the joint nonlinear effects can be neglected. Here we assume that the BJ is fastened enough and undergoes low amplitude dynamic loading.

Investigation is carried out on a structure consisting of two identical steel beams as shown in Fig. 1. The joint is held tight by a M8 bolt-nut pair. In the initial FE model, the nodes located at the joining surface were merged together. This model is developed using the finite element software ANSYS utilizing 8-node 3-D elements (solid185). The accelerometers' mass loading effects are included as point masses on their mounting spots.

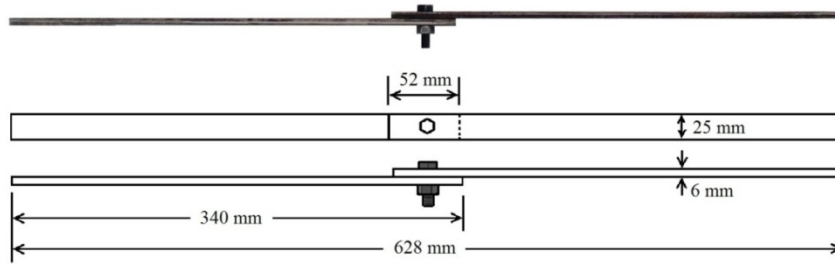


Fig. 1. Dimensions of the bolted beam structure

The first six bending frequencies of the initial model are listed in Table 1. Comparing these results with their experimental counterparts, one notices higher values for the numerically predicted modes. This shows much higher joint stiffness than the actual value. So, it is concluded that the structure is considered much stiffer than in reality and the flexibility effect of the joint is not included.

To deal with this problem, it is assumed that the bolted joint effects are extended to a region in both sides of the contact interface named as the ‘bolted joint affected region (BJAR)’ shown schematically in Fig. 2. A new methodology is proposed to model this region named the ‘contact zone element (CZE)’ technique. The difference between this technique and the IE method can be realized from Fig. 2. This figure shows a lap joint model of two metal beams. As shown in the figure, in the IE method, a thin layer of unknown material is considered between the common surfaces of the substructures. In the CZE technique, some portions of the elements comprising the substructures on both sides of the interface are used.

There are three advantages for the CZE technique compared with the IE approach. Firstly, the thickness of the structure is increased by the amount of the interface layer thickness whereas in the CZE approach, the thickness doesn’t change. Secondly, generating the interface layer in large structural models requires reproducing the whole model. In contrast, in the contact zone model, it is only necessary to apply little modifications in the substructures. Thirdly, in the interface layer method, the mass of the structure is changed whereas in the CZE method, the density of the elements can be chosen the same as the neighboring elements.

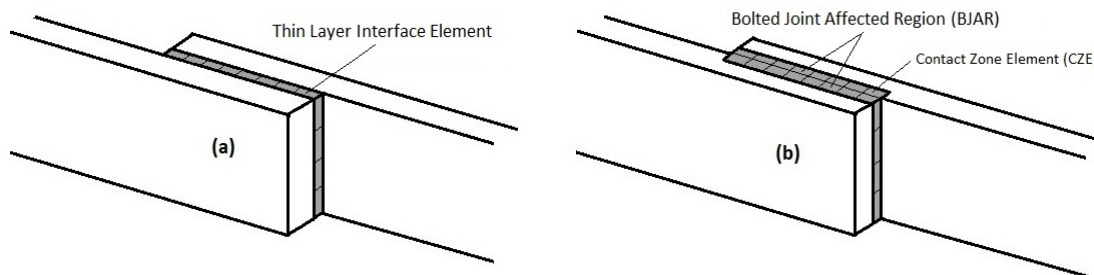


Fig. 2. Comparison of two techniques; (a) IE approach, (b) BJAR approach

It is necessary to implement proper constitutive relations for joint interface modeling with the CZE technique. Constitutive relations do not represent the fundamental laws of the nature.

Instead, they can be considered as a definition of the ideal material usually called the constitutive models. Such constitutive models must have the following properties [11]:

- They must be capable to reproduce the important features of the joint response.
- There must be a systematic trend to deduce the model parameters from experimental data.
- Integration of the model into a structural-level FE code must be practical.

If we have the stress and strain vectors in the form of:

$$\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{yz} \quad \tau_{zx}\}^T \quad (1)$$

$$\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{zx}\}^T \quad (2)$$

then we can cast the CZE constitutive relations into:

$$\{\sigma\} = [C]\{\varepsilon\} \quad (3)$$

$$[C] = \begin{bmatrix} [C_{nn}] & [C_{ns}] \\ [C_{sn}] & [C_{ss}] \end{bmatrix} \quad (4)$$

Equation 3 describes the Hook's law and is valid only for small strain values. In Eq. 4, $[C]_{6 \times 6}$ is the CZE constitutive matrix and relates the stresses to the strains. $[C_{nn}]_{3 \times 3}$ is the normal sub-matrix, $[C_{ss}]_{3 \times 3}$ is the shear sub-matrix and finally, $[C_{ns}]_{3 \times 3}$ and $[C_{sn}]_{3 \times 3}$ contain the coupling effects between the shear and normal behaviors within the interface. In this work, the coupling effects are neglected. Therefore, Eq. 4 will be rearranged into:

$$[C] = \begin{bmatrix} [C_{nn}] & [0] \\ [0] & [C_{ss}] \end{bmatrix} \quad (5)$$

The stiffness matrix adoption for the CZE technique can be performed basically the same as the ongoing routines for solid elements. Thus, the stiffness matrix is obtained by:

$$[k] = \int_V [B]^T [C] [B] dV \quad (6)$$

where $[B]$ is the strain-displacement matrix.

Referring to Eq. 5, the constitutive matrix for the CZE technique in a linear elastic behavior can be drawn as in Eq. 7:

$$[C] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1}{2}-\nu & 0 & 0 \\ & & & & \frac{1}{2}-\nu & 0 \\ sym & & & & & \frac{1}{2}-\nu \end{bmatrix} \quad (7)$$

In fact, Eq. 7 contains the stiffness matrix of an isotropic material and apparently, only the Young's Modulus E , and the Poisson's ratio ν are the two independent parameters sufficient for

describing the CZE behavior. Appropriate values of these parameters that are already related to the normal and shear stiffness values of the elements are unknown, and hopefully, can be identified by the FE model updating process. On the other hand, one can adopt suitable amounts for the CZE shear and normal stiffness values through adjustment of such parameters. In Eq. 8, the allowable ranges for the parameters to avoid meaningless and infinite matrix elements in [C] are depicted.

$$E > 0, -1 < \nu < 0.5 \tag{8}$$

This is only applicable to elastic isotropic materials. A negative Poisson’s ratio indicates lateral expansion on an stretched specimen. Such materials can be produced practically and are called auxetic materials. The other boundary limit 0.5 for ν is considering incompressible materials. Rubber-like materials exhibit this type of behavior.

Having a closer look at the CZE elasticity matrix [C] in Eq. 7, the following pattern can be realized:

$$\begin{bmatrix} \times & \times & \times & \vdots & & & & \\ \times & \times & \times & \vdots & & & & \\ \times & \times & \times & \vdots & & & & \\ \hline & & & \vdots & \times & & & \\ & & & \vdots & & \times & & \\ & & & \vdots & & & \times & \end{bmatrix}$$

where the X marks stand for numbers and the unoccupied spaces represent zero values in the matrix. The upper right and lower left sub-matrices present the relation between normal strains and shear stresses and also between shear strains and normal stresses. As they are vanishing, there is no coupling effect between these elements.

The constitutive relations are still worthwhile even though they only model the actual properties of the materials partially. They represent a mathematical model of a physical system based on limited experimental observations that let predict the system behavior under various situations. Obviously, the quality of predictions depends inherently on the skills of the test staff in proper idealization of the actual material so that one can figure out the experimental outcomes from a quantitative point of view. The case is especially more fatal in modeling BJ interfaces because there exist several uncertainties in their dynamical behavior.

In this work, the predominant and significant characteristics of the BJ linear behavior is used in spite of their complicated nature. The CZE is a 8-noded type element and its demeanor assimilates the rest of the elements of the model except their elastic properties are different from the neighboring elements. This element can be easily exploited in commercial FE codes. Fig. 3 illustrates such a model. It is clear from the figure that the CZE thickness is equal to the neutral planes offset of the two bolted beams at the beginning of the optimization process. In addition, their density is corresponding to those of the nearby elements, structural steel here, at the moment.

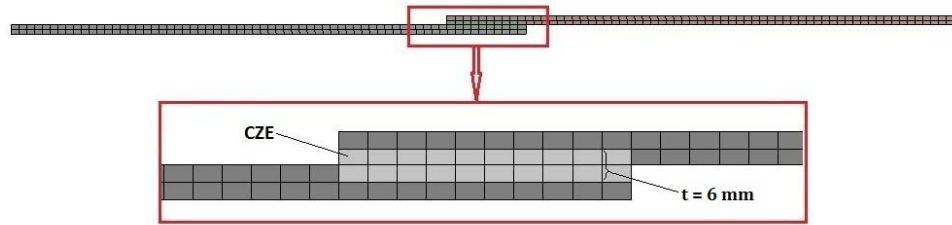


Fig. 3. Simulation of bolted joint interface with the BJAR approach

3. Modal experiment

To specify the parameters of the system's dynamical behavior such as natural frequencies, the physical structure went through a modal examination and then FRFs were precisely extracted. Fig. 4 shows the test set-up for measuring the dynamic response of the structure. The structure consists of two similar steel beams for which the free-free supporting condition is simulated using two strings about one meter length.

Excitation was done by a modal hammer and the exerted force was measured through the sensor mounted on the hammer tip. Five impacts were applied at each excitation point to ensure reliability. To receive the structure vibrational response, two accelerometers were stuck on both sides of the contact zone symmetrically (Points A and B in Fig. 4). The excitation points were selected with respect to points A and B. Load-cells and accelerometers record data in the time domain which is converted into the frequency domain using an appropriate software. Having divided the structural response to the input excitation in the frequency domain, the FRF of the structure can be obtained. Fig. 5 illustrates the FRF of the structure for the interval 0 to 1600 Hz which encompasses the excitation and response at point B. The first six bending natural frequencies of the structure are listed in Table 1.



Fig. 4. Test set-up

Table 1. Test frequencies and the corresponding values of initial FE model

Mode	Test frequency (Hz)	Initial FE model frequency (Hz)	Error (%)
1	72.14	80.07	11.0
2	207.85	208.67	0.4
3	386.29	433.59	12.2
4	657.34	690.62	5.1
5	970.84	1084.60	11.7
6	1334.69	1440.40	7.9

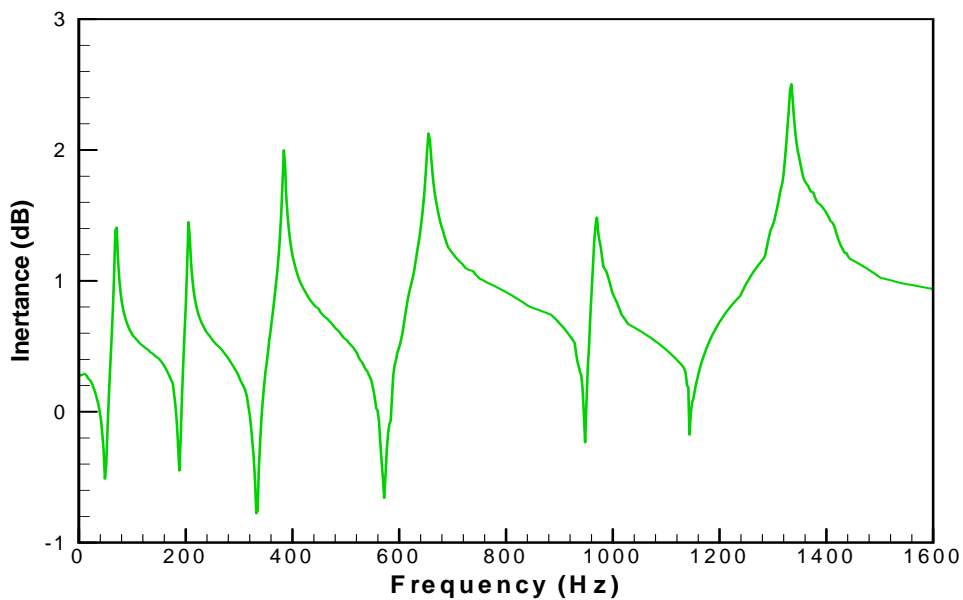


Fig. 5. Frequency response function of the structure

The behavior of the CZE technique is influenced by some unknown elasticity parameters which in turn affect the linear response of the structure. By the model updating procedure using experimental data, proper amounts of these parameters are identified and dynamic behavior of the model becomes similar to that observed by the real structure.

4. Identification of parameters

Identification of joint parameters is performed by the Direct Optimization (DO) module of the ANSYS software. After defining the desired parameters in ANSYS, an objective function is also

defined as a solver in MS-Excel and the connection between the two softwares is carried out through the parameters of the objective function. In this stage, an optimization procedure is executed using a Multi Objective Genetic Algorithm (MOGA) where the objective function is defined as the difference between the first four bending natural frequencies obtained from the analytical and experimental modal analyses to be minimized. The fifth and sixth bending natural frequencies are also reserved in order to evaluate the accuracy of the model prediction approach for higher frequencies. This objective function is defined in the following equation.

$$\min \sum_{j=1}^4 w_j \left[\frac{\omega_j^{(a)}}{\omega_j^{(e)}} - 1 \right]^2 \quad \text{with} \quad w_j \geq 0 \quad \text{and} \quad \sum_{j=1}^4 w_j = 1 \quad (9)$$

In Eq. 9, $\omega_j^{(a)}$ and $\omega_j^{(e)}$ are the analytical and experimental natural frequencies respectively and w_j are the weighting factors corresponding to each mode. Since in the present research, there is no preferable mode, all weighting factors are set equal to unity. In each iteration step of the optimization procedure, the eigenvalues are first computed and then the objective function is determined. The computed objective functions are compared to the values of the former step and the iteration process continues up to minimize the objective function. The eigenvalues are then computed based on the following equation:

$$(K - \lambda_i M)\phi = 0, \lambda_i = \omega_i^2, \quad i = 1, 2, \dots, 6 \quad (10)$$

where K and M are the structural stiffness and mass matrices respectively. λ and ϕ are the eigenvalues and eigenvectors respectively.

As can be seen from Fig. 6, a zero displacement constraint is defined along the z direction for the model to avoid changing the order of the mode shapes when changing the design parameters in the optimization process.

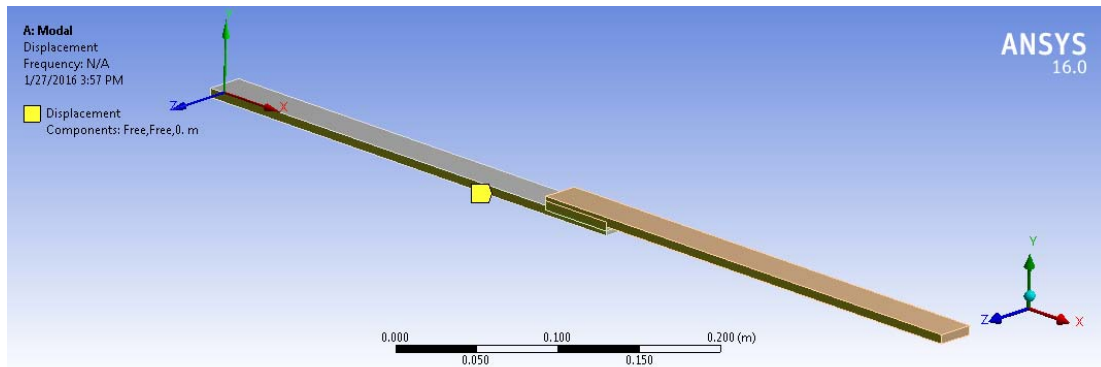


Fig. 6. Zero displacement constraint in z direction

This constraint does not let any structural displacement along the z direction, therefore, any transverse bending (in the z-x plane) or torsion can be avoided. This guarantees all the modes to

be of bending type in the x-y plane for the desired frequency range. On the other hand, since the defined parameters of the model and their variations along with the updating process is local, the order of the modes doesn't change. The beam's axial modes are also sufficiently far from the frequency range under consideration.

Initial values for the updating parameters of the CZEs are chosen to be as structural steel properties (neighboring elements' properties). Since the bolted joints cause a local reduction in the structural stiffness at the joint region, the allowable range for the Young's modulus of this region may be chosen to vary from 0.01 to 1.1 times the steel's modulus of elasticity. In order to prevent non-usual material properties, the Poisson's ratio is chosen in the limited interval $0 < \nu < 0.5$.

At the first stage of the identification process, the modulus of elasticity and the Poisson's ratio of the joint elements were chosen as the updating parameters. In this step, the thicknesses of the CZEs are set equal to the distance between the neutral planes of the two connecting beams and the density of the CZE was chosen to be the density of structural steel. By considering the results of Table 2 it can be seen that among the first four natural frequencies which were included in the model updating process, all modes are corrected except the third mode. However, the prediction of this model for the non-updated frequencies (the fifth and sixth frequencies) are not so well and there is a considerable difference between theory and experiment at these frequencies.

In the second step, in order to increase the accuracy of model prediction, the thickness of CZEs t was added to the updating parameters. The amounts of natural frequencies after minimization of the objective function are given in Table 2. As it can be seen from table 2, in addition to error reduction of the third natural frequency, the fifth and sixth natural frequencies are also predicted more accurately than the previous predictions.

In the third step, the density of the CZEs ρ was added to the updating parameters as well as the thickness of the layer. The amounts of natural frequencies obtained by minimization of the objective function are given in Table 2. Given the results presented by Table 2, it is clear that much better results are obtained in this case and the fifth and sixth natural frequencies, which were not included in the optimization process, are now predicted with a good accuracy. The identified values for the parameters in this case are given in Table 3. As it is clear, the Young's moduli of the CZEs are reduced nearly one ninth of their initial values. In Fig. 7, a schematic view of the BJAR in the final model is indicated after determining its proper thickness.

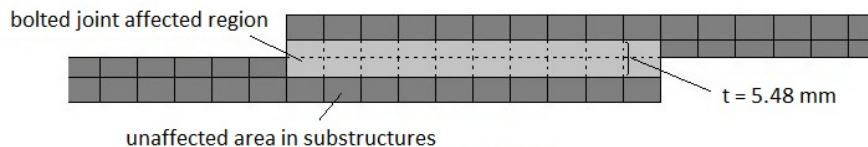


Fig. 7. Thickness of the BJAR after model updating

Table 2. A comparison of natural frequencies observed from the test and predicted by the updated FE model

Mode	Test frequency (Hz)	After updating					
		(E and ν)		(E, ν and t)		(E, ν , t and ρ)	
		Frequency (Hz)	Error (%)	Frequency (Hz)	Error (%)	Frequency (Hz)	Error (%)
1	72.14	71.92	-0.3	72.76	0.86	72.06	-0.11
2	207.85	206.90	-0.46	207.91	0.03	206.99	-0.41
3	386.29	406.78	5.3	398.43	3.14	390.03	0.97
4	657.34	655.90	-0.22	643.57	-2.1	656.25	-0.16
5	970.84	1020.10	5.1	1005.80	3.6	987.72	1.74
6	1334.69	1383.90	3.69	1356.50	1.63	1335.30	0.04

The history of the objective function minimization process is indicated in Fig. 8. As it can be seen from this figure, using the genetic algorithm, the solution converges in 6 iterations. In each iteration, fifty design points are evaluated.

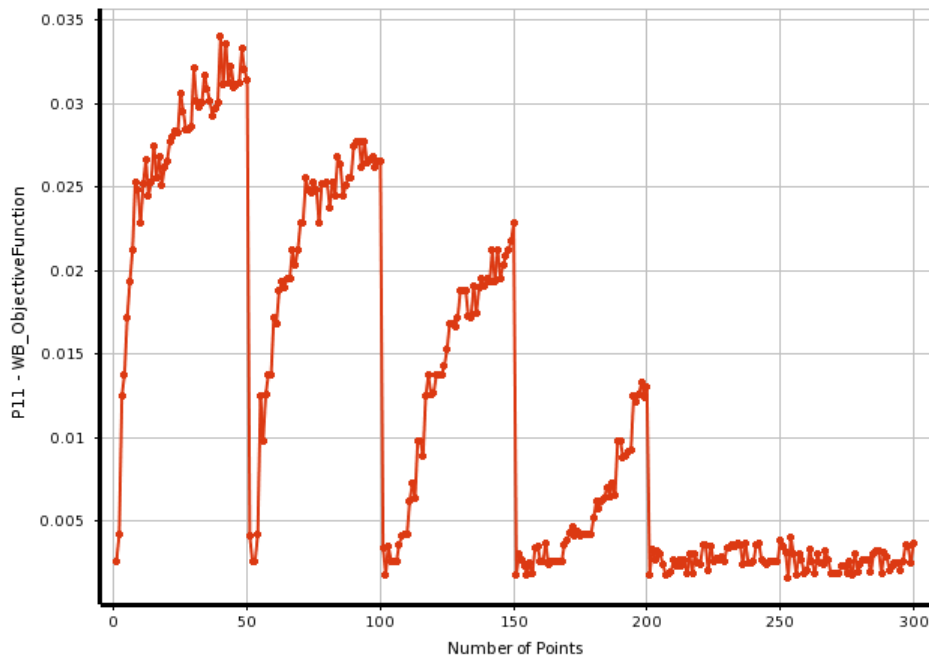


Fig. 8. The history chart of minimizing the objective function

Table 3. Optimum values of BJAR parameters in the updated model

Parameter	Young's modulus (GPa)	Poisson's Ratio	Shear modulus (GPa)	Thickness (mm)	Density (kg/m ³)
Optimum value	19.93	0.196	8.33	5.48	2257.3

5. Discussion and conclusion

Obtaining a simple model which can simulate the dominating physics of a bolted joint is very important in modeling of large and complex structures. In this paper, a new approach called CZE was introduced for simulation of bolted joint affected regions. The linear dynamics of a bolted joint was modeled using these elements and parameters of this model are identified by minimizing an objective function. For evaluation of the first six bending natural frequencies of the system, modal testing has been executed and the structure's frequency response function between 0 to 1600 Hz was extracted. Parameters associated with the elastic behavior of CZE, like the Young's modulus and the Poisson's ratio were used in the first step of the updating process. Due to poor prediction of higher frequencies, in the second step, thickness of the CZE was used as an updating parameter in addition to the Young's modulus and the Poisson's ratio. In the third step, both the thickness and density were used as updating parameters to predict the higher natural frequencies more accurately. The results from the third step have the best agreement with experimental measurements in comparison with the other steps. The proposed method in this paper has an efficient performance for predicting the higher mode frequencies which don't participate in model updating. It can be concluded that the quality of bolted joints interface simulation depends on parameters like the thickness and density of the CZE in addition to elasticity properties and in fact, there is an optimum thickness and density for the BJAR which are determined by the model updating process. The Young's modulus of CZE is approximately one-ninth of its initial value. This indicates the effects of a bolted joint based on structural stiffness reduction in the joint region. The numerical results indicate a good agreement with modal measurements and the updated model predicts the experimental results properly.

The proposed model has three advantages as compared with the common interface layer element method. Firstly, the thickness of the structure is increased by the amount of the interface layer thickness whereas in CZE approach, the thickness doesn't change. Secondly, generating the interface layer in large structural models requires reproducing the whole model. In contrast, in the contact zone model, it is only necessary to apply a little change in the substructures. Thirdly, in the interface layer method, the mass of the structure is changed whereas in the CZE method, the density of the elements can be chosen the same as the neighboring elements. These features are of great importance especially in large structures which have a lot of joints and in order to obtain more accurate results, the density can be considered as a design parameter. In this paper, in the case where the density is not used as a design parameter, the results are in acceptable range and only for obtaining more accurate results at the third step of model updating, the density is considered as a design parameter. However, this is not a requirement in general.

The presented model in this paper can be used in model updating of joints for large and complex structures using the commercial FEM softwares like ANSYS.

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