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A one-dimensional model for variations of longitudinal wave velocity under different thermal conditions

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Ultrasonic testing is a versatile and important nondestructive testing method. In many industrial applications, ultrasonic testing is carried out at relatively high temperatures. Since the ultrasonic wave velocity is a function of the workpiece temperature, it is necessary to have a good understanding of how the wave velocity and test piece temperature are related. In this paper, variations of longitudinal wave velocity in the presence of a uniform temperature distribution or a thermal gradient is studied using one-dimensional theoretical and numerical models. The numerical model is based on finite element analysis. A linear temperature gradient is assumed and the length of the workpiece and the temperature of the hot side are considered as varying parameters. The workpiece is made of st37 steel, its length is varied in the range of 0.04-0.08 m and the temperature of the hot side is changed from 400 K to 1000 K. The results of the theoretical model are compared with those obtained from the finite element model (FEM) and very good agreement is observed.

1. Introduction

Ultrasonic nondestructive testing (UT) is widely used in research and industry. Its use is not only limited to detection of flaws, but it can also be used for characterization of mechanical and metallurgical properties of materials. The velocity of ultrasonic waves is a function of temperature. Therefore, by measuring the ultrasonic wave velocity, one can measure the workpiece temperature.

Hayashi et al. [1] measured the temperature dependence of the velocity of sound in liquid Pb, Sn, Ge and Si in the ranges of 610–1078 K, 608–1463 K, 1215–1443 K, and 1723–1888 K, respectively. They concluded that in both liquid Pb and liquid Sn, the velocities of sound decrease, linearly with increasing temperature. However, in Ge, the velocity of sound has a distinct maximum around 1280 K and decreases linearly at higher temperatures, and in Si, the

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velocity of sound increases monotonically with increasing temperature in the investigated temperature range. Tsai et al. [2] made a temperature sensor with high accuracy and quick response time for measuring the air temperature. Nowacki and Kasprzyk [3] measured the velocities of longitudinal and transverse waves in X90CrMoV18 and X14CrMoS17 in the temperature range of 293-1173 K. By using these wave velocity measurements, they were able to find the relationship between the value of elastic constants and temperature. Periyannan and Balasubramaniam [4] used various ultrasonic wave-guides to measure the temperature of a heat treatment furnace.

In this paper, the effect of a uniform temperature and a unidirectional thermal gradient on longitudinal ultrasonic wave velocity in a specimen made from st37 steel is investigated. To evaluate the effect of temperature on velocity, a mathematical model is developed. Furthermore, a numerical model is also developed for this problem using finite element method. In both models, the effects of changes in two parameters of workpiece length and hot side temperature are considered.

2. Heat transfer formulation

To study the effect of a thermal gradient on the wave velocity in a specimen, the boundary conditions of the specimen should be first specified. We assume that the heat transfer takes place in one direction, and the two endpoints of the specimen have two different temperatures. Ultrasonic wave velocities are calculated under these conditions. Heat is transferred from the lower end of the test piece to its upper end. Assuming the heat transfer to take place in a steady state, the temperatures of lower and upper ends of the test piece are designated as T_1 and T_2, respectively as shown in Fig. 1.

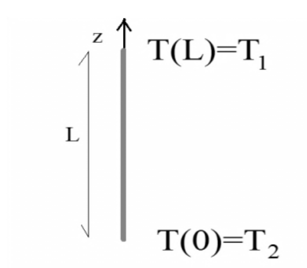

Fig. 1: Unidirectional thermal boundary conditions

The unidirectional heat transfer equation is [5]:

$$\frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

By integrating this equation, we get,

$$T = az + b \tag{2}$$

According to Fig. 1, the boundary conditions are as follows:

$$T(L) = T_1 \tag{3}$$

$$T(0) = T_2 \tag{4}$$

Substituting these boundary conditions into Eq. 2 gives,

$$b = T_2 \tag{5}$$

$$a = -\frac{T_2 - T_1}{L} \tag{6}$$

Therefore,

$$T = -\left(\frac{T_2 - T_1}{L}\right) z + T_2 \tag{7}$$

To calculate the temperature gradient, we use the following equation,

$$\frac{dT}{dz} = -\frac{T_2 - T_1}{L} \tag{8}$$

T_1 and T_2 are directly measurable, and the following equation is used to calculate the length of the work piece, L_f, in the presence of a thermal gradient,

$$dL = -\alpha\, z \left(\frac{T_2 - T_1}{L}\right) dz \tag{9}$$

$$L_f = L + \frac{\alpha}{2} L(T_2 - T_1) \tag{10}$$

In equations 9 and 10, α is the coefficient of thermal expansion of the specimen.

3. Velocity equation in the presence of thermal gradient

By dividing the specimen into thin layers and assuming a constant temperature in each of these layers, we can write,

$$\Delta t = \sum_{i=1}^{N} \Delta t_i \tag{11}$$

where Δt is half of the time difference between two consecutive echoes and Δt_i is half of the time interval the wave travels a distance Δz_i (see Fig. 2). The number of layers in the z-direction is N.

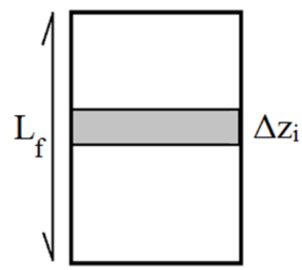

Fig. 2: Δz_i in the presence of a thermal gradient

The temperature is constant along Δz_i and therefore the velocity is also constant along Δz_i. This velocity is designated as C_i. The velocity measured by the transducer in the presence of a thermal gradient is called C_m. According to Eq. 11, we can write,

$$\frac{L_f}{C_m} = \sum_{i=1}^{N} \frac{\Delta z_i}{C_i} \tag{12}$$

The intervals Δz_i are very small and their total number tends to infinity, therefore, Eq. 12 can be written as follows,

$$\frac{L_f}{C_m} = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{\Delta z_i}{C_i} = \int_0^{L_f} \frac{dz}{C(z)} \tag{13}$$

The temperature-dependent velocity C_T can also be written as [6],

$$C_T = C_1 \sqrt{\left(1 + \frac{\beta}{E_1}(T - T_1)\right)(1 + 3\alpha(T - T_1))} \tag{14}$$

In Eq. 14, C_1, E_1 and β are the one-dimensional longitudinal wave velocity at T_1, the Young's modulus at T_1 and the specific parameter of the temperature effect on the Young's modulus respectively. Based on Eqs. 7 and 14, the wave velocity can be written as a function of z as follows,

$$C(z) = C_1\sqrt{(1-\frac{\beta a}{E_1}(L-z))(1-3\alpha a(L-z))} \tag{15}$$

By using Eqs. 13 and 15, the following integral equations are obtained,

$$\frac{L_f}{C_m} = \frac{1}{C_1}\int_0^{L_f}\frac{dz}{\sqrt{(1-\frac{\beta a}{E_1}(L-z))(1-\ 3\alpha a(L-z))}} \tag{16}$$

$$C_m = \frac{C_1 L_f}{\int_0^{L_f}\frac{dz}{\sqrt{(1-\frac{\beta a}{E_1}(L-z))(1-\ 3\alpha a(L-z))}}} \tag{17}$$

To solve this integral, the following change of variable is considered,

$$A = -\frac{3\beta\alpha a^2}{E_1} \tag{18}$$

$$B = 3\alpha a\left(1-\frac{\beta aL}{E_1}\right)+\frac{(1-3a\alpha L)\beta a}{E_1} \tag{19}$$

$$C = \left(1-\frac{\beta aL}{E_1}\right)(1-3a\alpha L) \tag{20}$$

and the average wave velocity in the specimen is obtained as,

$$C_m = \frac{\sqrt{A}C_1L_f}{tan^{-1}\left(\frac{\sqrt{A}\left(L_f-\frac{B}{2A}\right)}{\sqrt{-AL_f{}^2+BL_f+C}}\right)+tan^{-1}(\frac{B}{2\sqrt{AC}})} \tag{21}$$

4. Analysis

To study the effects of temperature and thermal gradient on the one-dimensional longitudinal wave velocity, we use Eqs. 14 and 21. The parameters used in the model are listed in Table 1. The values of α, β, and E_1 are taken from [6].

Table 1: Parameters used in the model of the st37 specimen

Parameter	**Parameter value**
T_1 (K)	300
C_1 (m/s) – 300K	5048.8
α (1/K)	11.6×10^{-6}
β (Pa/K)	-40.6×10^{6}
E_1 (Pa)	200.1×10^{9}

4.1. The effect of temperature on C_T

To study the effect of temperature on C_T, we consider Eq. 14. C_T is independent of the length of the specimen which is assumed to be constant and equal to 0.05 m. The temperature-dependent velocity, C_T, at different temperatures is given in Table 2.

Table 2: The effect of temperature on C_T

Temperature (K)	C_T (m/s)
400	5006.01
500	4962.48
600	4918.2
700	4873.15
800	4827.31
900	4780.65
1000	4733.15

According to Table 2, with the increase of temperature, C_T decreases almost linearly.

4.2. The effect of work piece length on C_m

By assuming constant temperatures on the two sides of the specimen (T_1 = 300 K, T_2 = 400 K), the effect of specimen length on longitudinal wave velocity C_m is investigated. The specimen length is arbitrarily chosen between 0.04-0.08 m. The values obtained for C_m for different specimen lengths are given in Table 3.

Table 3: The effect of work piece length on the velocity C_m and thermal gradient

L (m)	C_m (m/s)	***a*** (K/m)
0.04	5027.44	-2500
0.05	5027.44	-2000
0.06	5027.44	-1667
0.07	5027.44	-1429
0.08	5027.44	-1250

The mean value of longitudinal wave velocities at temperatures 300 K and 400 K is 5027.4 m/s, which has negligible difference with C_m. According to Table 3, C_m can be considered independent of the length of the workpiece. Furthermore, there is no one-to-one correspondence between a and C_m.

4.3. The effect of hot side temperature on C_m

To study the effect of temperature T_2, we consider T_1 to be constant and equal to 300 K. The specimen length is also considered to be constant and equal to 0.05 m. Based on these assumptions, the wave velocity in the specimen is calculated for different values of T_2 (see Fig. 3 and Table 4).

Table 4: The effect of T_2 on C_m and $(C_{T1}+ C_{T2})/2$

T_2 (K)	C_m (m/s)	$(C_{T1}+ C_{T2})/2$ (m/s)
400	5027.44	5027.4
500	5005.81	5005.64
600	4983.88	4983.5
700	4961.66	4960.98
800	4939.13	4938.06
900	4916.27	4914.73
1000	4893.1	4890.98

According to Fig. 3, as T_2 increases, C_m decreases almost linearly and the difference between C_m and C_{T2} increases. Furthermore, the difference between C_m and $(C_{T1}+ C_{T2})/2$ slightly increase as T_2 increases.

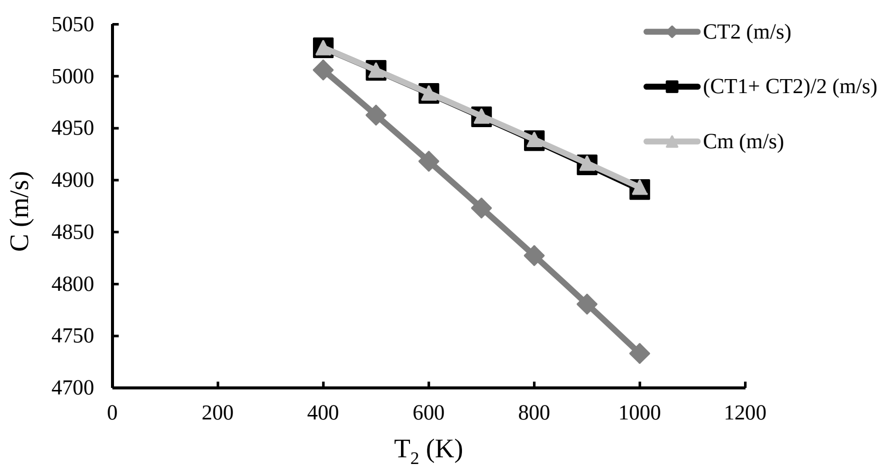

Fig. 3: The effect of T_2 on C_m, C_{T2} and $(C_{T1}+ C_{T2})/2$

5. Finite element analysis

To model the process of sending and receiving a wave and applying the thermal conditions, the finite element (FE) software package ANSYS is used. First, the effect of temperature on C_T is investigated numerically and then, the effects of variations in L and T_2 on C_m are studied. Finally, the numerical results are compared with theoretical results. The element size was set to 0.0125 mm in the FE model. The simulation process is carried out in two stages. In the first stage, the thermal condition of the specimen is considered. Element LINK33 is used to simulate this stage. The thermal boundary conditions and the required properties of st37 steel are input into the software and the distribution of temperature is found. Fig. 4 shows the temperature distribution in the presence of the thermal gradient.

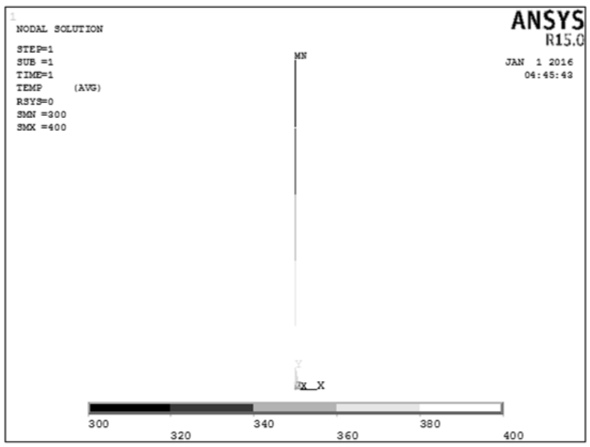

Fig. 4: Temperature distribution obtained from thermal simulation in the presence of a thermal gradient

The second stage deals with structural simulation of the specimen where element LINK180 is used. In the second stage, the temperature distribution obtained from the first stage is input into the model as an external load. The ultrasonic pulse is transmitted into the test piece from its upper surface and received at the same location. The shape of the ultrasonic pulse is shown in Fig. 5.

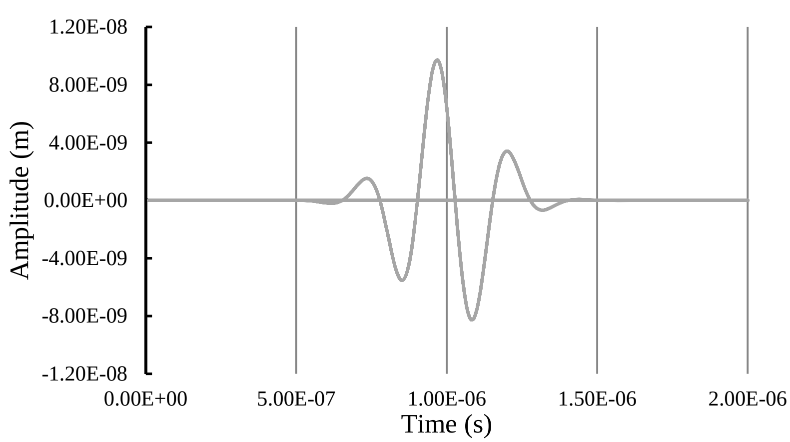

Fig. 5: The ultrasonic pulse used in the simulation process

The velocity C_m is calculated by dividing the length of the workpiece (multiplied by two) by the time difference between two successive echoes. Figure 6 shows the effect of temperature on C_T obtained from theory and FE modeling.

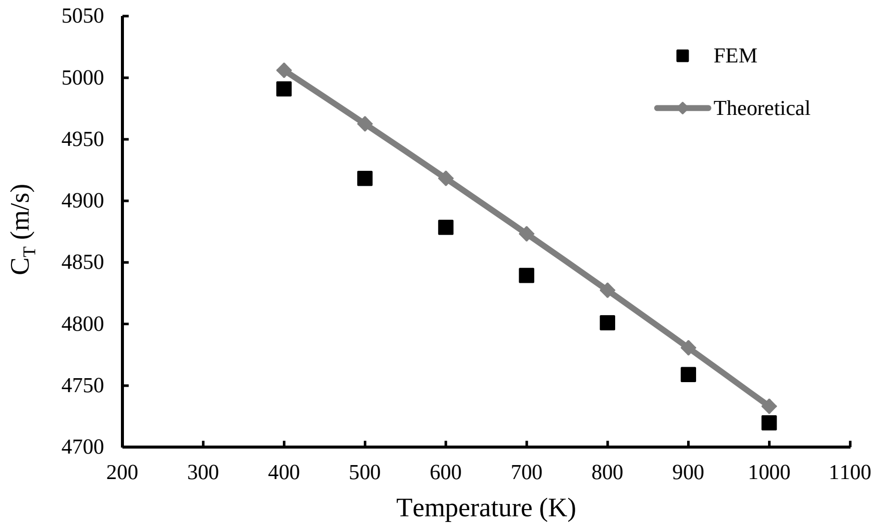

Fig. 6: The effect of temperature on C_T obtained from theory and FE modeling.

Figure 7 and Table 5 compare the effect of variations in L on C_m in the finite element and theoretical models.

Table 5: The effect of variations in L on C_m in the finite element and theoretical models

L (m)	$C_{m\text{-Theoretical}}$ (m/s)	$C_{m\text{-FEM}}$ (m/s)	***Error*** (%)
0.04	5027.44	5002.9	0.49
0.05	5027.44	5005.4	0.44
0.06	5027.44	5009.2	0.36
0.07	5027.44	5011.8	0.31
0.08	5027.44	5012.3	0.3

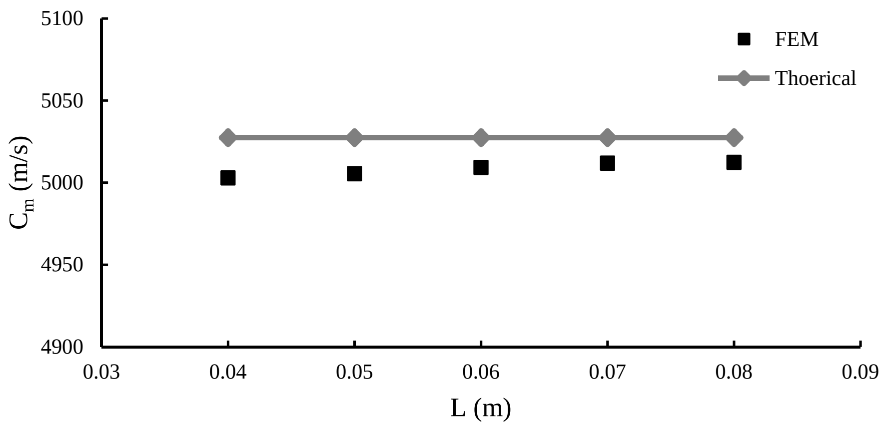

Fig 7: The effect of variations in L on C_m in the finite element and theoretical methods

The velocity $C_{m\text{-Theoretical}}$ is independent of the part's length L but the velocity $C_{m\text{-FEM}}$ slightly increases and approaches a certain value with increase in L. The reason for this can be attributed to the element type LINK180 that has been used in our finite element model. In ANSYS, the cross section of this element should be input to the model [7]. By increasing the length to cross section ratio ($L/A \rightarrow \infty$), the velocity $C_{m\text{-FEM}}$ tends to the designated value mentioned above. This value is very close to $C_{m\text{-Theoretical}}$.

Fig. 8 and Table 6 compare the effect of variations in T_2 on C_m in the finite element and theoretical methods.

Table 6: The effect of variations in T_2 on C_m in the finite element and theoretical models

T_2 (K)	$C_{m\text{-FEM}}$ (m/s)	$C_{m\text{-Theoretical}}$ (m/s)	***Error*** (%)
400	5002.9	5027.44	0.49
500	4988.33	5005.81	0.35
600	4971.4	4983.88	0.25
700	4952.15	4961.66	0.19
800	4935.5	4939.13	0.07
900	4919	4916.27	0.06
1000	4902.59	4893.1	0.19

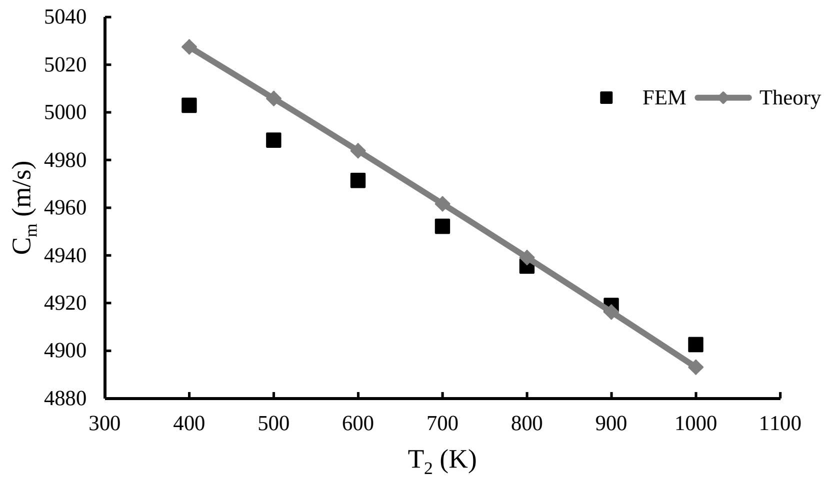

Fig 8: The effect of variations in T_2 on C_m in the finite element and theoretical models.

Mean difference of 0.5% between the two sets of results shows good agreement between the theoretical and finite element models. Based on the theoretical and finite element results, the following observations can be made:

- As uniform temperature increases, C_T decreases almost linearly.
- As T_2 increases, C_m decreases almost linearly.
- As T_2 increases, the difference between C_m and C_{T2} increases.
- As T_2 increases, the difference between C_m and $(C_{T1}+ C_{T2})/2$ slightly increases.
- C_m is independent of L, and it depends only on the temperatures T_1 and T_2 and material properties of the workpiece.
- According to Table 3, by increasing L, $C_{m\text{-Theoretical}}$ remains unchanged, but the thermal gradient a decreases; therefore, there is no one-to-one correspondence between a and $C_{m\text{-Theoretical}}$.

6. Conclusions

In this paper, the effect of the presence of a uniform temperature and a thermal gradient in a workpiece on ultrasonic wave velocity is studied. First, a one-dimensional mathematical model is developed based on wave and heat transfer equations. The effect of two parameters of workpiece length and hot side temperature are studied on this model as two important parameters of the thermal gradient. Then, a finite element model is developed for the same problem. The results obtained from the mathematical and finite element models are compared and good agreement is observed. Using this model, velocity measurements can be done more precisely in the presence of a uniform temperature and a thermal gradient.

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