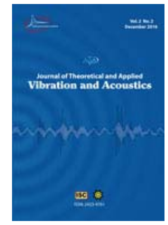
Journal of Theoretical and Applied Vibration and Acoustics 2(2) 119-132 (2016)

I S A V

Journal of Theoretical and Applied

Vibration and Acoustics

journal homepage: http://tava.isav.ir



Spectrally formulated finite element for vibration analysis of an Euler-Bernoulli beam on Pasternak foundation

Ali Mokhtari, Vahid Sarvestan, Hamid Reza Mirdamadi *

Department of Mechanical Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

ARTICLE INFO

Article history:
Received 14 February 2016

Received in revised form
5 June 2016

Accepted 23 June 2016

Available online 27 July 2016

Keywords:
Foundation stiffness

Shear layer stiffness

Spectral finite element model

Divergence instability

Wave domain analysis

ABSTRACT

In this article, vibration analysis of an Euler-Bernoulli beam resting on a Pasternak-type foundation is studied. The governing equation is solved by using a spectral finite element model (SFEM). The solution involves calculating wave and time responses of the beam. The Fast Fourier Transform function is used for temporal discretization of the governing partial differential equation into a set of ordinary differential equations. Then, the interpolating function for an element is derived from the exact solution of governing differential equation in the frequency domain. Inverse Fourier Transform is performed to rebuild the solution in the time domain. The foremost advantages of the SFEM are enormous high accuracy, smallness of the problem size and the degrees of freedom, low computational cost and high efficiency to deal with dynamic problems and digitized data. Moreover, it is very easy to execute the inverse problems by using this method. The influences of foundation stiffness, shear layer stiffness and axial tensile (or compressive) forces on the dynamic characteristic and divergence instability of the beam are investigated. The accuracy of the present SFEM is validated by comparing its results with those of classical finite element method (FEM). The results show the ascendency of SFEM with respect to FEM in reducing elements and computational effort, concurrently increasing the numerical accuracy.

Nomenclature

EI	flexural rigidity ($N \cdot m^2$)	N	total number of frequency samples
$f(x,t)$	excitation force (N)	P_x	constant axial force (N)
f_c	cut-off frequency (Hz)	$Q(x,t)$	shear force (N)

* Corresponding Author.
Email address: hrmirdamadi@cc.iut.ac.ir (H. R. Mirdamadi)

http://dx.doi.org/10.22064/tava.2016.20910

k	wave number $(\mathrm{rad/m})$	$[S^e]$	elemental dynamic stiffness matrix
k_s	stiffness of the shear layer (N)	V_{wave}	wave speed or phase speed $(\mathrm{m/s})$
k_w	Winkler foundation modulus $(\mathrm{N/m^2})$	ρA	mass per length of beam $(\mathrm{kg/m})$
L	span between two end supports (m)	ρI	mass moment of inertia per length $(\mathrm{kg \cdot m})$
L^e	length of an element (m)	ω	sampling frequency $(\mathrm{rad/s})$
$M(x,t)$	bending moment $(\mathrm{N \cdot m})$	ω_c	cut-off frequency $(\mathrm{rad/s})$

1. Introduction

In many engineering applications, beams are located on elastic foundations; for example, structures on elastic medium that constitute parts of any machinery for isolation functions, concrete structures on soil in civil engineering applications and the cases in railway applications. Due to such applications, dynamic behavior of beams on elastic foundations is a subject of technological interest. During the past decades, a variety of models have been presented for beams resting on elastic foundations such as soil etc. Those structures supported along their main axis are represented by numerous approaches such as Winkler, Pasternak or Vlasov, Flonenko - Borodich foundations. The Winkler modeling, one of the most fundamental methods, was recommended in 1867 by Winkler. The model presents a linear algebraic association between the normal displacement of the structure and the contact pressure [1]. The Winkler elastic foundation has a single parameter that was used in the studies made by Shin *et al.* [2], Hsu [3], De Rosa [4], and Lee *et al.* [5]. The Winkler model represents the soil medium by a set of equally independent spring elements. This method gives effortlessness in finding closed-form solutions [6, 7]. In addition, it grants the possibility of obtaining a nonlinear behavior with lower computational struggle compared to other methods [8-13]. There are several studies on the Winkler elastic foundation modeling in literature. Zhou [14] and Eisenberger [15] studied a general solution to vibrations of beams on a variable Winkler elastic foundation. Auersch [16] accomplished a study about infinite beams on half-space compared with finite and infinite beams on a Winkler support. Eisenberger and Clastornik [17] inspected the vibrations and buckling of a beam on a variable Winkler elastic foundation. Gupta *et al.* [18] provided buckling and vibration behavior of polar orthotropic circular plates with linearly varying thickness. Furthermore, Ruge and Birk [19] examined the dynamic behavior of infinite beam models, devoting value to asymptotic behavior at high frequencies. Oz and Pakdemirli [20] has scrutinized the resonances of shallow beams resting on elastic foundations. The classification of the two parameter foundation models with shear modulus or transverse modulus is recognized as a Pasternak foundation model. In two parameter foundation models, the first parameter of the foundation is still the Winkler elastic foundation parameter. The Pasternak foundation model is introduced by Shin *et al* [2], El-Mously [21], Zhu and Leung [22], Arboleda-Monsalve *et al.* [23], Ma *et al.* [24] and Civalek [25].

The solutions of equation of motion for a beam resting on Pasternak-type foundation were obtained by various solution techniques including the perturbation method [21], the classical finite element method (FEM) [22, 26] and the Laplace transform method [27].

FEM has been the most popular method in many fields of engineering and science [28, 29]. This method may present accurate dynamic characteristics of a structure if the wavelength is large compared to the mesh size. The FEM matrices are typically formulated from presumed frequency-independent polynomial shape functions. Nevertheless, as the most important negative aspect of FEM, it is recognized that a large number of elements should be used to get dependable results particularly at higher frequencies. Clearly, this prerequisite may raise the time and cost of computations. Consequently, many researchers ponder alternatives to the FEM. Recently, the Spectral Finite Element Method (SFEM) based on the Fast Fourier Transform (FFT) has been broadly used in dynamic analysis of structures. In this method, the equations of system are solved in the frequency domain and the FFT is utilized to convert the time domain responses to the wave domain and vice versa. SFEM utilizes the exact solution of governing differential equation (if available) in the wave domain as the interpolating function for element formulation. Hence, SFEM, in contrast with FEM, represents a structural member with much lower number of elements regardless of its dimensions and without the necessity to separate the member into large number of refined elements for increased accuracy. Narayanan and Beskos [30] founded the basic concepts of SFEM. To the authors' knowledge, Doyle [31] is one of the early studies which employ SFEM to wave propagation in structures. Doyle [32], and Doyle and Farris[33] formulated SFEM for elementary isotropic waveguides and Gopalakrishnan *et al*. [34] utilized SFEM for higher-order waveguides. Lee *et al*. [35], Kim *et al*. [36] and some other researchers applied SFEM to dynamic problems of beams, plates and trusses. Roy Mahapatra and Gopalakrishnan [37] used SFEM for the analysis of axial-flexural-shear coupled wave propagation in laminated composite thick beams. Vinod *et al*. [38] applied SFEM for free vibration and wave propagation analysis of uniform and tapered rotating beams. Lee and Lee [39] exploited SFEM for an extended Timoshenko beam. Sarvestan *et al*. [40] used SFEM for vibration analysis of cracked viscoelastic Euler-Bernoulli beam subjected to a moving load.

In the literature reviewed so far, spectrally formulated finite element solution of Euler-Bernoulli beams on a two parameter elastic foundation has not been considered and this is the most remarkable innovation of this study. Moreover, to the best of authors' knowledge, wave speed, cut-off frequency, and closed-form of divergence instability of this structure has not yet been investigated in the literature. Thus, the main objectives of this article are: (1) to develop the SFEM for a transversely vibrating beam by considering the Euler-Bernoulli beam theory, elastic foundation and axial tensile (or compressive) force at the same time, (2) to highlight the fewer energy and time for discretization and more acceptable numerical accuracy of this model as compared with those of FEM, and (3) to investigate the effects of elastic foundation and axial tensile force on the vibration and wave characteristics (*i.e.* the natural frequencies, wave speed and cut-off frequency) and the divergence instability.

2. Theory

The governing wave differential equation of an Euler-Bernoulli beam resting on Pasternak foundation, under tensile force (see Fig. 1), is given as [24, 41]:

$$EI\frac{\partial^4 w}{\partial x^4} - (P_x + k_s)\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} + k_w w = f(x,t) \tag{1}$$

where $w(x,t)$ is the lateral deflection, $f(x,t)$ is the excitation force, EI is the flexural rigidity, P_x is the axial pretension, ρA is the mass per length of beam, L is the length of the beam, L^e is the length of the element, k_w is the Winkler foundation modulus and k_s is the stiffness of the shear layer.

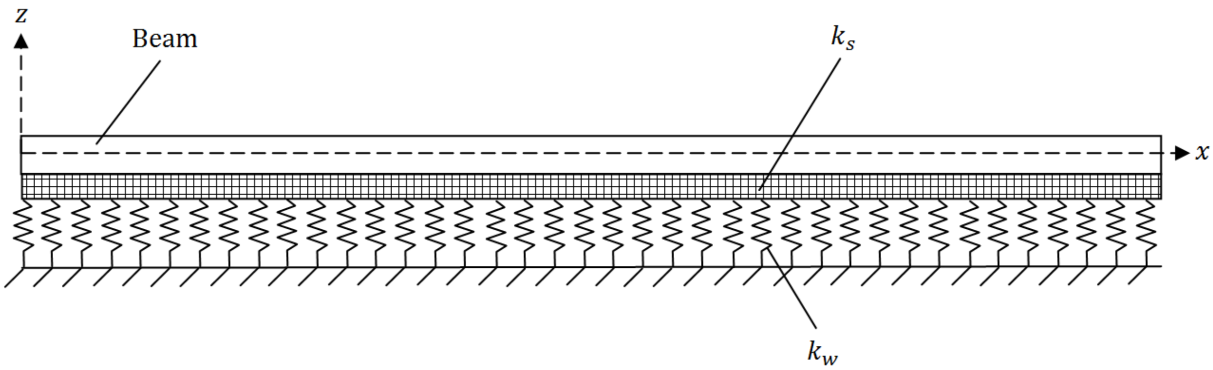


Fig. 1. Schematic illustration of a beam resting on a Pasternak foundation

The force boundary conditions are given as:

$$\begin{cases} Q(0,t) = -Q_{n\_1}(t) - P_x \dfrac{\partial w(0,t)}{\partial x} \\ M(0,t) = -M_{n\_1}(t) \\ Q(L^e,t) = Q_{n\_2}(t) - P_x \dfrac{\partial w(L^e,t)}{\partial x} \\ M(L^e,t) = M_{n\_2}(t) \end{cases} \tag{2}$$

where $Q(x,t)$ and $M(x,t)$ are the shear force and bending moment defined as:

$$Q(x,t) \triangleq -EI \frac{\partial^3 w}{\partial x^3} \tag{3}$$

$$M(x,t) \triangleq EI \frac{\partial^2 w}{\partial x^2} \tag{4}$$

$M_{n\_1}(t)$ and $Q_{n\_1}(t)$ are the bending moment and the lateral shear force applied at $x = 0$. $M_{n\_2}(t)$ and $Q_{n\_2}(t)$ are the bending moment and the lateral shear force applied at $x = L^e$ (see Fig. 2).

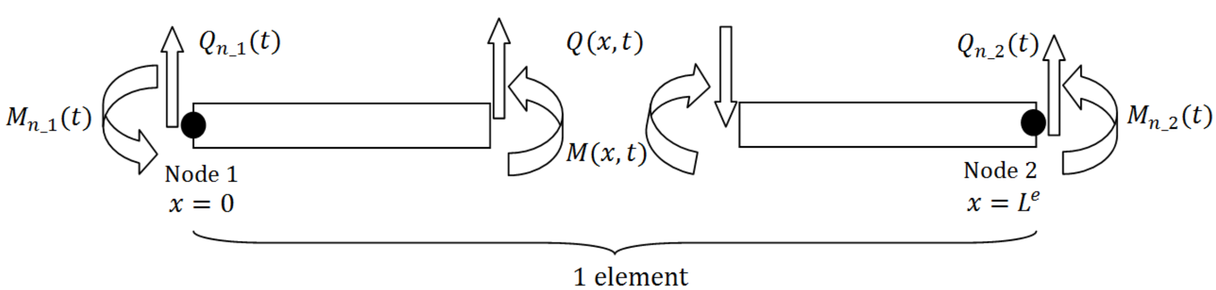


Fig. 2. A finite element model of the beam

3. Temporal and spatial discretization

Assume that the transverse displacement of an Euler-Bernoulli beam in the spectral form is [42]:

$$w(x,t) = \frac{1}{N}\sum_{n=0}^{N-1} W_n(x;i\omega_n)e^{i\omega_n t} \tag{5}$$

where $W_n(x;i\omega_n)$ are the spectral components of the displacement field $w(x,t)$. Also, N is the total number of frequency samples used in DFT transformation. By using Eq. (5), for a specific discrete frequency $\omega = \omega_n$, keeping in mind that the subscript is vanished for more legibility, Eq. (1) can be converted into wave-domain expression as:

$$EI\frac{d^4W}{dx^4} - (P_x + k_s)\frac{d^2W}{dx^2} + (-\omega^2\rho A + k_w)W = F \tag{6}$$

where F represents the spectral components of $f(x,t)$. Forced boundary conditions given in Eqs. (3) and (4) are similarly transformed as:

$$\mathcal{M} = EI\frac{d^2W}{dx^2} \tag{7}$$

$$\mathcal{Q} = -EI\frac{d^3W}{dx^3} \tag{8}$$

The exact interpolating functions obtained from solving ODEs considering free bending (flexural) vibration, *i.e.* Eq. (6) without F, are:

$$W = C_w e^{-ikx} \tag{9}$$

where k is the wavenumber. Substituting the interpolating functions into the homogeneous equations, resulting from ODEs, *i.e.* Eq. (6), yields a dispersion relation as:

$$EIk^4 + (P_x + k_s)k^2 + k_w - \omega^2\rho A = 0 \tag{10}$$

Therefore, $\{C_w\} = \{C_{w1} \quad C_{w2} \quad C_{w3} \quad C_{w4}\}^T$ are the constants to be derived from the boundary conditions at the two nodes (see Fig. 3), i.e. nodal displacements $\{u^e\} = \{W_{n\_1} \quad \theta_{n\_1} \quad W_{n\_2} \quad \theta_{n\_2}\}^T$ where $W_{n\_1} \triangleq W|_{x=0}$, $\theta_{n\_1} \triangleq \left.\frac{dW}{dx}\right|_{x=0}$, $W_{n\_2} \triangleq W|_{x=L^e}$, and $\theta_{n\_2} \triangleq \left.\frac{dW}{dx}\right|_{x=L^e}$.

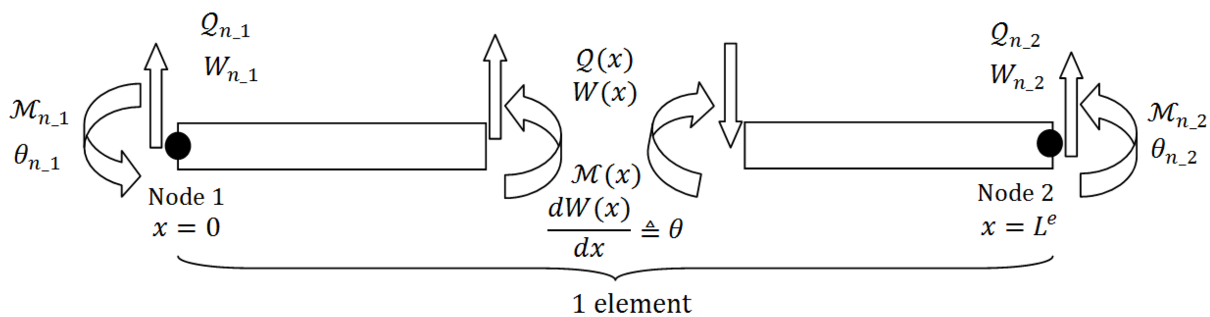


Fig. 3. An SFE of the beam with equivalent nodal forces and the degrees of freedom (DOFs)

Thus, nodal displacement and unknown constants can be related as:

$$\{u^e\} = \begin{Bmatrix} W_{n\_1} \\ \theta_{n\_1} \\ W_{n\_2} \\ \theta_{n\_2} \end{Bmatrix} = [T]\{C_w\} \tag{11}$$

from the force boundary conditions Eqs. (2), (7) and (8), nodal forces $\{F^e\} = \{\mathcal{Q}_{n\_1} \quad \mathcal{M}_{n\_1} \quad \mathcal{Q}_{n\_2} \quad \mathcal{M}_{n\_2}\}^T$ and unknown constants can be related as:

$$\{F^e\} = \begin{Bmatrix} \mathcal{Q}_{n\_1} \\ \mathcal{M}_{n\_1} \\ \mathcal{Q}_{n\_2} \\ \mathcal{M}_{n\_2} \end{Bmatrix} = [T']\{C_w\} \tag{12}$$

where $\mathcal{Q}_{n\_1} = -\mathcal{Q}|_{x=0} - P_x \frac{dW}{dx}\Big|_{x=0}$, $\mathcal{M}_{n\_1} = -\mathcal{M}|_{x=0}$, $\mathcal{Q}_{n\_2} = \mathcal{Q}|_{x=L^e} + P_x \frac{dW}{dx}\Big|_{x=L^e}$ and $\mathcal{M}_{n\_2} = \mathcal{M}|_{x=L}$.

Finally, substituting $\{C_w\}$ from the nodal displacement vector, i.e. Eq. (11) into the nodal force vector, i.e. Eq. (12), the nodal displacements and nodal forces are interrelated as:

$$\{F^e\} = [T'][T]^{-1}\{u^e\} = [S^e]\{u^e\} \tag{13}$$

Here, $[S^e]$ is the elemental dynamic stiffness matrix for the beam. The assembled global structural system equations for the SFEM can be obtained by adopting the assembly procedure as commonly used in the conventional FEM. The computations of the assembled global matrices for $r = 0, 1, \dots, n-1$ are done numerically. These equations can be solved to derive the nodal displacement vector, $\{u\}$, for known nodal forces. Consequently, global nodal displacement $\{u\}$ can be substituted into the inverse FFT algorithm to obtain the time domain responses.

4. Asymptotic analysis

The wave speed or phase speed is defined as:

$$V_{wave} = Re\left(\frac{\omega}{k}\right) \tag{14}$$

it should be noted that the phase speed is defined with respect to real wavenumber k. As a result, the speeds change with frequencies. The value of the wave speed depends on frequency and there can be a frequency after which the wavenumbers transit from being purely imaginary to complex or real wavenumbers resulting in propagation of the wave mode. This transition frequency is called the cut-off frequency. Thus, in Eq. (14) when $Re(k) \to 0$ (or $V_{wave} \to \infty$), an asymptotic equation of Eq. (10) for the natural frequency parameter ω can be obtained as:

$$\omega_c = \sqrt{\frac{k_w}{\rho A}} \quad or \quad f_c = \frac{1}{2\pi}\sqrt{\frac{k_w}{\rho A}} \tag{15}$$

where ω_c is the cut-off frequency. The cut-off frequency is a function of Winkler foundation parameter k_w and the geometrical properties of the beam. If $k_w = 0$, there is no possibility for a cut-off frequency.

5. Divergence instability

When the beam is under compressive force $-P_x$, the fundamental natural frequency becomes zero and the beam becomes unstable by divergence. Thus, the critical load at which the divergence instability (divergence load P_{x_D}) happens can also be achieved by bearing in mind the existence of the non-trivial equilibrium position, in other words, the static eigenvalue problem. For a simply supported Euler-Bernoulli beam resting on a Pasternak-type foundation, the static eigenvalue problem can be derived unswervingly from Eq. (10) by inserting $\omega = 0$ as follows:

$$EIk^4 + (P_x + k_s)k^2 + k_w = 0 \tag{16}$$

Eq. (16) gives four roots as:

$$\begin{aligned} k_1 = -k_2 &= \sqrt{\frac{-(P_x + k_s) + \sqrt{(P_x + k_s)^2 - 4EIk_w}}{2EI}} = \mathbb{K}_1 \\ k_3 = -k_4 &= \sqrt{\frac{-(P_x + k_s) - \sqrt{(P_x + k_s)^2 - 4EIk_w}}{2EI}} = \mathbb{K}_2 \end{aligned} \tag{17}$$

Thus, the non-trivial equilibrium displacement can be written as:

$$W = C_{w1}e^{-i\mathbb{K}_1 x} + C_{w2}e^{i\mathbb{K}_1 x} + C_{w3}e^{-i\mathbb{K}_2 x} + C_{w4}e^{i\mathbb{K}_2 x} \tag{18}$$

Applying the simply supported boundary conditions to Eq. (18) gives:

$$\mathbb{K}_1 = \frac{n\pi}{L} \quad or \quad \mathbb{K}_2 = \frac{n\pi}{L} \qquad (\, n = 1, 2, 3, \ldots) \tag{19}$$

Lastly, substituting Eq. (19) into Eq. (16) gives the closed-form of the divergence load as:

$$P_{x\_D} = -\frac{k_w L^4 + EI(n\pi)^4 + k_s(n\pi L)^2}{(n\pi L)^2} \qquad (\, n = 1, 2, 3, \ldots) \tag{20}$$

6. Numerical results and discussion

For numerical studies, the numerical values of material and geometrical parameters of simply supported Euler-Bernoulli beam are: length $L = 4$ m, Young's modulus $E = 2.1 \times 10^{11}$ GPa, cross-sectional second moment of inertia $I = b \times h^3/12\ m^4$, width $b = 0.3$ m, height $h = 0.3$ m and mass density $\rho = 7860\ \mathrm{kg/m^3}$. The non-dimensional properties of the Pasternak foundation are $\overline{k_w} \triangleq \frac{k_w L^4}{EI} = 10$ and $\overline{k_s} \triangleq \frac{k_s L^2}{EI} = 25$ unless otherwise mentioned. P_x is 40 kN unless otherwise mentioned. The impulsive load is subjected to the mid-span of the simply supported beam that has 100 kN amplitude and the duration of 0 to 1.1 ms. At first, SFEM is used to study the natural frequencies, divergence instability and the phase speed. Next, SFEM is used to simulate time responses in the same structure. Numerical examples have been conducted to evaluate the accuracy and efficiency of the SFEM solutions through a comparison with those from FEM. For SFEM wave domain analysis, the whole length of the beam is divided into one SFE and for SFEM time domain analysis, the same length is divided into two SFEs. On the other hand, for FEM dynamic analysis, the total number of FEs used in the analysis is varying in order to improve the numerical accuracy.

6.1. Natural frequencies and divergence instability

The SFEM is evaluated by comparing the natural frequencies obtained from this method, those obtained from the analytical approach [42] and those from the FEM (see Table 1). Table 1 shows that the SFEM results are identical to the exact analytical results given by Usiklee [42] while the FEM results converge to the SEM results (obtained by using one SFE) as the total number of FEs used in FEM is increased. This implies that the SFEM could provide extremely accurate solutions even by using smaller number of FEs in the spectral-domain (one SFE) as compared with those obtained from FEM. This is true especially at high frequency modes. From Table 1, one may observe that the natural frequencies are in general increased as axial tension, Winkler foundation modulus and stiffness of the shear layer are increased.

The effect of compressive axial force on the fundamental natural frequencies of the simply supported Euler Bernoulli beam is exhibited in Fig. 4 when $\overline{k_w} = 10$ and $\overline{k_s} = 25$. It is obvious that the frequency decreases with compressive force increasing and this descent gets faster when the axial force gets closer to the divergence load. Finally, at the divergence load P_{x_D}, the first natural frequency becomes zero.

Table 1. Natural frequencies of the Euler-Bernoulli beam resting on Pasternak-type foundation

Investigated parameters	Method (number of elements)	Natural frequencies (Hz)			
		f_1	f_2	f_3	f_{10}
$P_x = 0$ kN	Theory [42]	43.95	175.79	395.52	4394.70
$\overline{k_w} = 0$	SEM (1)	43.95	175.79	395.52	4394.70
$\overline{k_s} = 0$	FEM (5)	43.95	176.08	398.66	5588.17
	FEM (30)	43.95	175.79	395.53	4398.26
$P_x = 40$ kN	Theory [42]	43.96	175.80	395.53	4394.71
$\overline{k_w} = 0$	SEM (1)	43.96	175.80	395.53	4394.71
$\overline{k_s} = 0$	FEM (5)	43.96	176.09	398.67	5588.18
	FEM (30)	43.96	175.80	395.54	4398.27
$P_x = 40$ kN	SEM (1)	46.16	176.36	395.78	4394.73
$\overline{k_w} = 10$	FEM (5)	46.16	176.65	398.92	5588.19
$\overline{k_s} = 0$	FEM (30)	46.16	176.36	395.79	4398.29
$P_x = 40$ kN	SEM (1)	83.80	225.10	447.97	4450.04
$\overline{k_w} = 10$	FEM (5)	83.80	225.34	450.79	5634.57
$\overline{k_s} = 25$	FEM (30)	83.80	225.10	447.98	4453.56

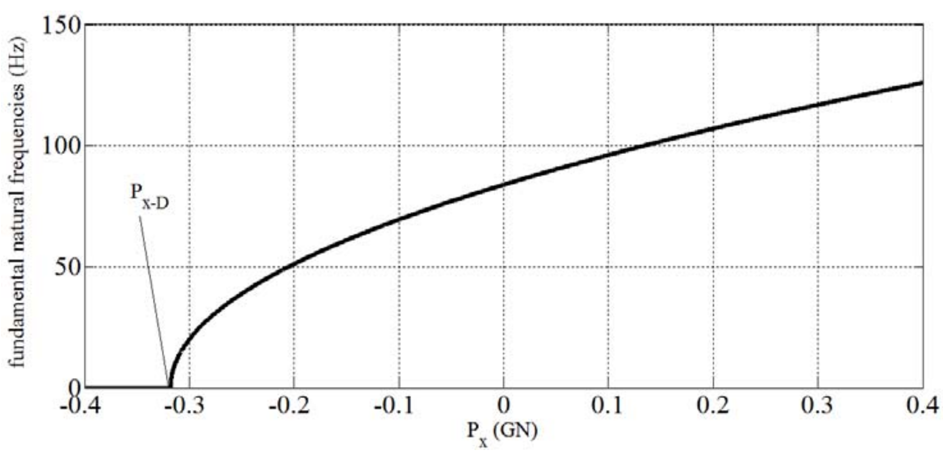


Fig. 4. Fundamental natural frequency versus axial force curve when $\overline{\mathrm{k_w}} = 10$ and $\overline{\mathrm{k_s}} = 25$

6.2. Wave speed

For the Euler-Bernoulli beam resting on Pasternak-type foundation, the wave speed variation with frequency is shown in Fig. 5. The effects of the Pasternak foundation is shown in Fig. 5 for $P_x = 40$ kN and $\overline{k_s} = 25$. The effects of Pasternak foundation show that the wave will have a cut-off frequency. In accordance with Eq. (15), cut-off frequency decreases as the radius of beam increases. Moreover, the cut-off frequency increases with the stiffness of the Winker foundation parameter.

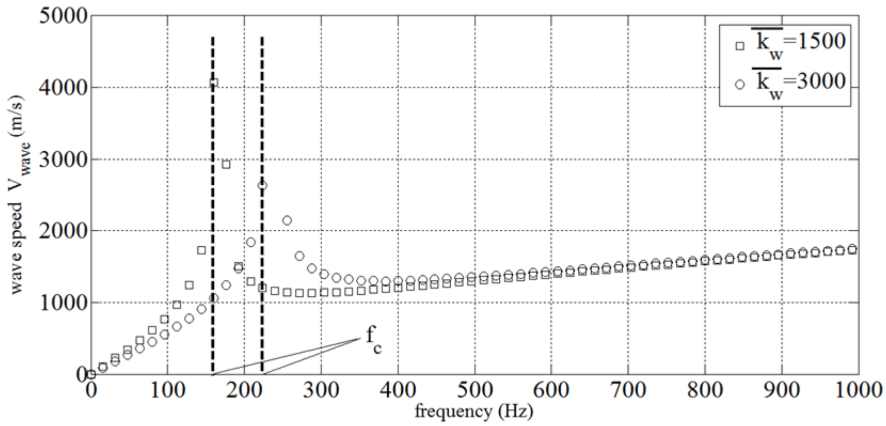


Fig. 5. Wave speed dispersion with wave frequency in Euler-Bernoulli beam resting on Pasternak-type foundation when $P_x = 40$ kN and $\overline{k_s} = 25$.

6.3. Time domain analysis

The structure jumps at divergence instability or buckling where its amplitude increases exponentially with time. In other words, when the axial force P_x crosses the critical value P_{x_D}, the first eigenfrequency becomes zero implying a divergence instability. Thus, the undeformed equilibrium configuration of the beam becomes linearly unstable. Consequently, Fig. 6 illustrates the divergence instability of the structure when $P_x = P_{x\_D}$, $\overline{k_w} = 10$ and $\overline{k_s} = 25$. In addition, this figure demonstrates that the results predicted by FEM would converge to those predicted by the present SFEM. It should be noted that the total number of FEs used in FEM could increase beyond 100. This implies that, in contrast to the FEM, the SFEM provides highly accurate results by using only a small number of finite elements (two SFEs).

From Table 1, the structure should have more stiffness and fundamental frequency for higher axial tension, Winkler foundation modulus and stiffness of the shear layer (see Figs. 7-9 respectively). Therefore, narrower time periods and lower amplitudes of time responses in Figs. 7-9 would demonstrate this fact of the Euler-Bernoulli beam resting on Pasternak-type foundation. It should be noted that, the time responses are obtained from SFEM.

Numerical results for two values of axial tension, $P_x = 40$ kN and 100 MN, is included in Fig. 7. As seen, one can generally say that the magnitude of deflection and time period of lower axial tension is larger than those of the higher one.

Numerical results obtained for the two values of Winkler foundation modulus, $\overline{k_w} = 0$ and 10, are compared in Fig. 8. It is found that the magnitude of deflection and time period of lower modulus is larger than those of higher modulus.

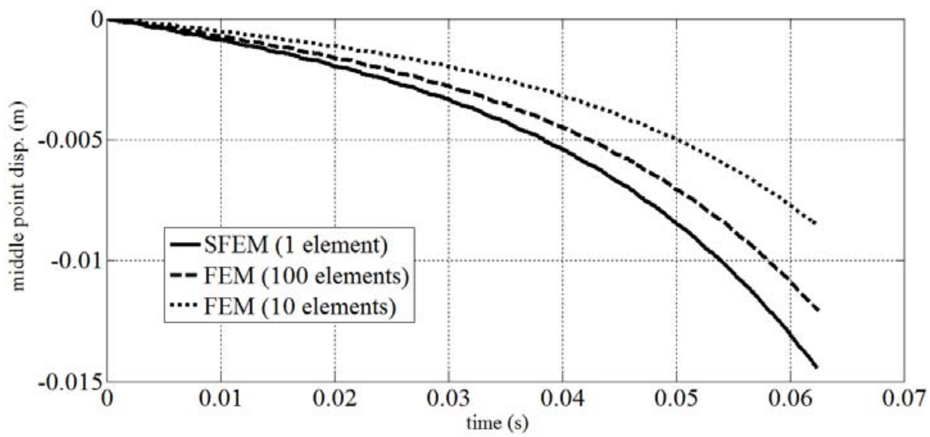


Fig. 6. Comparison of SFEM and FEM time response of the Euler-Bernoulli beam resting on Pasternak-type foundation when $\mathrm{P_x} = \mathrm{P_{x\_D}}$, $\overline{\mathrm{k_w}} = 10$ and $\overline{\mathrm{k_s}} = 25$

Numerical results obtained for the two values of stiffness of the shear layer $\overline{k_s} = 0$ and 25 are compared in Fig. 9. It shows that the amplitude of deflection and time period of the lower stiffness is larger than those of the higher stiffness.

One can conclude that higher axial tension, Winkler foundation modulus, and stiffness of the shear layer provide safer margins for design of the foundation and the beam.

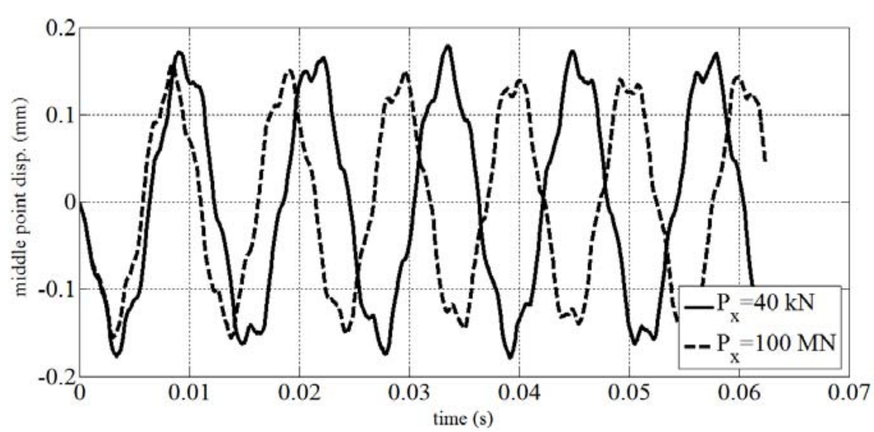


Fig. 7. The effects of tensile force on time responses when $\overline{\mathrm{k_w}} = 10$ and $\overline{\mathrm{k_s}} = 25$

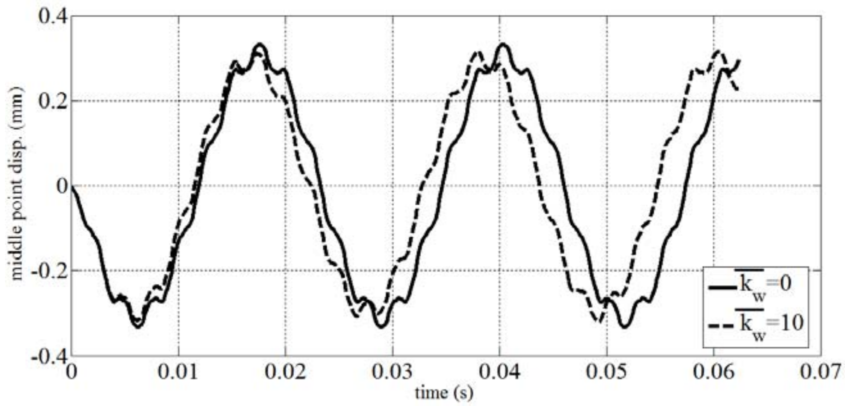


Fig. 8. The effects of Winkler foundation modulus on time responses when $P_x = 40$ kN and $\overline{k_s} = 0$

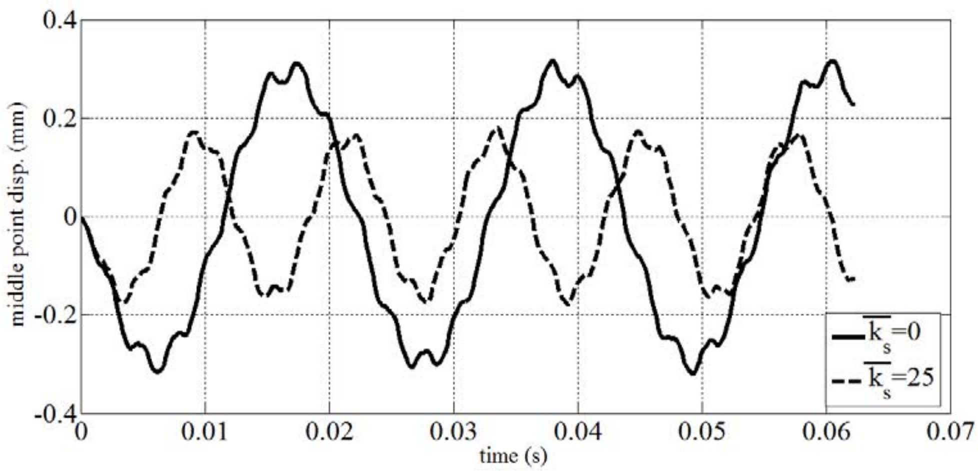


Fig. 9. The effects of stiffness of the shear layer on time responses when $P_x = 40$ kN and $\overline{k_w} = 10$

7. Conclusion

In this article, the spectrally formulated finite element model of an Euler-Bernoulli beam under axial tensile (or compressive) force resting on Pasternak-type foundation was presented. The main characteristic of SFEM in comparison with other methods such as FEM is demonstrated to be its higher accuracy together with using fewer elements. The SFEM method was shown to be an efficient alternative formulation instead of FEM or analytical analysis for wave propagation problems. Using the SFEM developed here is the main novelty of this article. Moreover, the effects of elastic foundation and axial tensile force on the vibration and wave characteristics (i.e. natural frequencies, wave speed and cut-off frequency) and divergence instability were investigated. Results obtained in the article give ideas about how the time and wave responses are affected by foundation parameters and axial forces. By the presented SFEM in this article, the vibration trend of a structure under axial force and resting on elastic foundation may be estimated.

References

[1] M.I. Gorbunov-Pasodav, T.A. Malikova, Design of structures on elastic foundations, (1973).
[2] Y. Shin, J. Yun, K. Seong, J. Kim, S. Kang, Natural frequencies of Euler-Bernoulli beam with open cracks on elastic foundations, Journal of Mechanical Science and Technology, 20 (2006) 467-472.
[3] M.H. Hsu, Vibration analysis of edge-cracked beam on elastic foundation with axial loading using the differential quadrature method, Computer Methods in Applied Mechanics and Engineering, 194 (2005) 1-17.
[4] M.A. De Rosa, Free vibrations of timoshenko beams on two-parameter elastic foundation, Computers & Structures, 57 (1995) 151-156.
[5] S.Y. Lee, Y.H. Kuo, F.Y. Lin, Stability of a Timoshenko beam resting on a Winkler elastic foundation, Journal of Sound and Vibration, 153 (1992) 193-202.
[6] K. Al-Hosani, S. Fadhil, A. El-Zafrany, Fundamental solution and boundary element analysis of thick plates on Winkler foundation, Computers & structures, 70 (1999) 325-336.
[7] M. Hetényi, Beams on elastic foundation; theory with applications in the fields of civil and mechanical engineering, University of Michigan Press, Ann Arbor, 1946.
[8] G. Gazetas, G. Mylonakis, Seismic soil-structure interaction: new evidence and emerging issues, Geotechnical Special Publication, 2 (1998) 1119-1174.
[9] K.P. Soldatos, A.P.S. Selvadurai, Flexure of beams resting on hyperbolic elastic foundations, International Journal of Solids and Structures, 21 (1985) 373-388.
[10] J.S. Smaill, Large deflection response of annular plates on Pasternak foundations, International Journal of Solids and Structures, 27 (1991) 1073-1084.
[11] K.K. Raju, G.V. Rao, Effect of a non-linear elastic foundation on the mode shapes in stability and vibration problems of uniform columns/beams, Journal of Sound and vibration, 160 (1993) 369-371.
[12] Y.S. Shih, P.T. Blotter, Non-linear vibration analysis of arbitrarily laminated thin rectangular plates on elastic foundations, Journal of Sound and Vibration, 167 (1993) 433-459.
[13] G.C. Tsiatas, Nonlinear analysis of non-uniform beams on nonlinear elastic foundation, Acta Mechanica, 209 (2010) 141-152.
[14] D. Zhou, A general solution to vibrations of beams on variable Winkler elastic foundation, Computers & Structures, 47 (1993) 83-90.
[15] M. Eisenberger, Vibration frequencies for beams on variable one-and two-parameter elastic foundations, Journal of Sound and Vibration, 176 (1994) 577-584.
[16] L. Auersch, Dynamic interaction of various beams with the underlying soil–finite and infinite, half-space and Winkler models, European Journal of Mechanics-A/Solids, 27 (2008) 933-958.
[17] M. Eisenberger, J. Clastornik, Vibrations and buckling of a beam on a variable Winkler elastic foundation, Journal of Sound and Vibration, 115 (1987) 233-241.
[18] U.S. Gupta, A.H. Ansari, S. Sharma, Buckling and vibration of polar orthotropic circular plate resting on Winkler foundation, Journal of Sound and Vibration, 297 (2006) 457-476.
[19] P. Ruge, C. Birk, A comparison of infinite Timoshenko and Euler–Bernoulli beam models on Winkler foundation in the frequency-and time-domain, Journal of Sound and Vibration, 304 (2007) 932-947.
[20] H.R. Öz, M. Pakdemirli, Two-to-one internal resonances in a shallow curved beam resting on an elastic foundation, Acta Mechanica, 185 (2006) 245-260.
[21] M. El-Mously, Fundamental frequencies of Timoshenko beams mounted on Pasternak foundation, Journal of Sound and Vibration, 228 (1999) 452-457.
[22] B. Zhu, A.Y.T. Leung, Linear and nonlinear vibration of non-uniform beams on two-parameter foundations using p-elements, Computers and Geotechnics, 36 (2009) 743-750.
[23] L.G. Arboleda-Monsalve, D.G. Zapata-Medina, J.D. Aristizabal-Ochoa, Timoshenko beam-column with generalized end conditions on elastic foundation: Dynamic-stiffness matrix and load vector, Journal of Sound and Vibration, 310 (2008) 1057-1079.

[24] X. Ma, J.W. Butterworth, G.C. Clifton, Static analysis of an infinite beam resting on a tensionless Pasternak foundation, European Journal of Mechanics-A/Solids, 28 (2009) 697-703.
[25] Ö. Civalek, Nonlinear analysis of thin rectangular plates on Winkler–Pasternak elastic foundations by DSC–HDQ methods, Applied Mathematical Modelling, 31 (2007) 606-624.
[26] A.G. Razaqpur, K.R. Shah, Exact analysis of beams on two-parameter elastic foundations, International Journal of Solids and Structures, 27 (1991) 435-454.
[27] E.B. Magrab, Natural frequencies and mode shapes of Timoshenko beams with attachments, Journal of Vibration and Control, 13 (2007) 905-934.
[28] J.J. Wu, A.R. Whittaker, M.P. Cartmell, The use of finite element techniques for calculating the dynamic response of structures to moving loads, Computers & Structures, 78 (2000) 789-799.
[29] L. Andersen, S.R.K. Nielsen, S. Krenk, Numerical methods for analysis of structure and ground vibration from moving loads, Computers & Structures, 85 (2007) 43-58.
[30] G.V. Narayanan, D.E. Beskos, Use of dynamic influence coefficients in forced vibration problems with the aid of fast fourier transform, Computers & Structures, 9 (1978) 145-150.
[31] J.F. Doyle, Wave propagation in structures, Springer-Verlag, New York, 1997.
[32] J.F. Doyle, A spectrally formulated finite element for longitudinal wave propagation, International Journal of Analytical and Experimental Modal Analysis, 3 (1988) 1-5.
[33] J.F. Doyle, T.N. Farris, A spectrally formulated finite element for flexural wave propagation in beams, International Journal of Analytical and Experimental Modal Analysis, 5 (1990) 99-107.
[34] S. Gopalakrishnan, M. Martin, J.F. Doyle, A matrix methodology for spectral analysis of wave propagation in multiple connected Timoshenko beams, Journal of Sound and Vibration, 158 (1992) 11-24.
[35] J. Kim, U. Lee, A.Y.T. Leung, Vibration analysis of the active multi-layer beams by using spectrally formulated exact natural modes, KSME International Journal, 15 (2001) 199-209.
[36] K. Moon-Young, K. Nam Ii, Y. Hee-Taek, Exact dynamic and static stiffness matrices of shear deformable thin-walled beam-columns, Journal of Sound and Vibration, 267 (2003) 29-55.
[37] D.R. Mahapatra, S. Gopalakrishnan, A spectral finite element model for analysis of axial–flexural–shear coupled wave propagation in laminated composite beams, Composite Structures, 59 (2003) 67-88.
[38] K.G. Vinod, S. Gopalakrishnan, R. Ganguli, Free vibration and wave propagation analysis of uniform and tapered rotating beams using spectrally formulated finite elements, International Journal of Solids and Structures, 44 (2007) 5875-5893.
[39] U. Lee, C. Lee, Spectral element modeling for extended Timoshenko beams, Journal of Sound and Vibration, 319 (2009) 993-1002.
[40] V. Sarvestan, H.R. Mirdamadi, M. Ghayour, A. Mokhtari, Spectral finite element for vibration analysis of cracked viscoelastic Euler–Bernoulli beam subjected to moving load, Acta Mechanica, 226 (2015) 4259-4280.
[41] T.M. Wang, J.E. Stephens, Natural frequencies of Timoshenko beams on Pasternak foundations, Journal of Sound and Vibration, 51 (1977) 149-155.
[42] U. Lee, Spectral element method in structural dynamics, John Wiley & Sons, 2009.