Journal of Theoretical and Applied Vibration and Acoustics 11 (2) 109-122 (2025)

I S A V

Journal of Theoretical and Applied

Vibration and Acoustics

journal homepage: http://tava.isav.ir

Influence of non-local parameter on period doubling behavior of CNTs conveying nanoparticles

Reza Ebrahimi [a,*], **Mohammad Sajjadnejad** [b]

[a] *Assistant Professor, Mechanical Engineering, Faculty of Engineering, Yasouj University, Yasouj 75918-74831, Iran*

[b] *Assistant Professor, Materials Engineering, Faculty of Engineering, Yasouj University, Yasouj 75918-74831, Iran*

Research Article

ARTICLE INFO

Article history:
Received 3 January 2025
Received in revised form
22 April 2025
Accepted 11 May 2025
Available online 11 August 2025

Keywords:
Period doubling
Carbon nanotubes
Nonlocal effect
Moving harmonic load
Winkler elastic foundation

ABSTRACT

In drug delivery systems, carbon nanotubes (CNTs) can be used as molecular channels to transport nanoparticles. In these applications, CNTs are subjected to a moving load, which leads to nonlinear vibrations of the CNTs. Therefore, the main goal of this study is to analyze the bifurcation behavior of CNTs under the effect of a moving harmonic load. For this purpose, modeling of the system has been done using the non-local Euler-Bernoulli beam theory and Winkler spring. The Galerkin approach and Runge-Kutta method have been used to discretize and solve the equation of motion, respectively. The effects of the moving load, elastic bed stiffness, and non-local elasticity parameter on the nonlinear dynamic response of the system have been investigated by using the bifurcation diagrams, phase plane, power spectrum, and Poincare sections. The results indicate various non-linear behaviors such as the jump phenomenon, the periodic, subharmonic, and quasi-periodic movements in the system response.

1. Introduction

Due to the unique physical, mechanical, and electrical properties of carbon nanotubes (CNTs), they are used in various nanotechnology applications such as drug delivery [1], biosensors [2], transistors [3], photocatalytic reactivity [4], etc. Associated with these applications, understanding the dynamic behavior of CNTs is very important [5-8]. CNTs are used in drug delivery systems as molecular channels with the aim of transporting nanoparticles. Therefore, CNTs carrying

---

[*] Corresponding author.

E-mail addresses: rebrahimi@yu.ac.ir (R. Ebrahimi), m.sajjadnejad@yu.ac.ir (M. Sajjadnejad).

http://dx.doi.org/10.22064/tava.2025.2049773.1259

nanoparticles are exposed to moving transverse loads that lead to transverse vibrations in them [9]. Based on this, many studies have been conducted on the vibration behavior of CNTs under moving load, some of which are mentioned below.

Simsek [10] has studied the forced vibrations of a carbon nanotube with a simple support under a moving harmonic load. Time domain responses have been obtained using modal analysis and the direct integration method. The effect of size ratio, speed, and excitation frequency of moving load on the dynamic responses of the system have been discussed. The results show that the dynamic deflections obtained from non-local solutions are greater than their corresponding classical solutions. For large values of the speed of the moving load, the dynamic deflections tend to zero, regardless of the excitation frequencies. Hong et al. [11] investigated the vibrations caused by the passage of a moving particle in a carbon nanotube placed on an elastic substrate. The behavior of the elastic bed is assumed to be linear. The time response has been calculated for different parameters. Pirmoradian et al. [12] proposed a nonlocal model to investigate the stability of a double-walled carbon nanotube (DWCNT) subjected to moving nanoparticles and embedded on an elastic substrate. All inertial terms of the moving nanoparticles were taken into account. The results indicated that considering van der Waals effects and increasing the stiffness of the elastic bed can improve the stability of the system. Sarparast et al. [13] studied the free and forced vibrations of a fluid-carrying nanotube exposed to a moving load in a thermomagnetic environment. Their results showed that the effect of moving load speed on the dynamic behavior of the system increases with the increase of surface residual stresses. Natsuki et al. [14] modeled the dynamic behavior of a double-walled nanotube embedded in an elastic medium under the passage of nanoparticles. In this model, the pipes and elastic bed are modeled as shells and a Winkler Spring, respectively. Also, van der Waals interaction forces have been taken into account. The results confirm that increasing the elastic coefficient of the bed reduces the dynamic deformation of the system. Also, with the increase in the speed of nanoparticles, the influence of the non-local parameter on the dynamic response of the system increases. Thongchom et al. [15] presented the vibration analysis of CNTs carrying nanoparticles based on the Timoshenko beam model, non-local elasticity, and the slip boundary condition. Ozmen and Esen [16] calculated the dynamic response of a CNT subjected to a magnetic field, a thermal field, and a moving load. The results showed that the increase of the doublet length scale parameter leads to a decrease in stiffness and consequently a reduction in the dimensionless frequency of the system. Ma et al. [17] derived the governing equations of a nanotube embedded in a vibrating medium under the action of moving nanoparticles. They showed that with the increase in the mass of the nanoparticle and consequently the increase in inertial effects, the difference between the results obtained from the moving nanomass with the moving nanoforce increases. Leng and Chang [18] proposed a mechanism for the transport of nanoparticles in a long CNT by using fluid-solid coupling and creating flow caused by temperature changes. Based on molecular dynamics simulations, they showed that a periodic temperature field can create a stable fluid flow and consequently transport nanoparticles inside the long CNT. Keshtkar et al. [19] investigated the dynamic response of a CNT carrying fluid and moving nanoparticles, considering the slip boundary conditions. The results showed that the natural frequency of the system decreases with the increase in fluid flow speed. Hashemian et al. [20] studied the dynamic stability of a nanobeam due to the passage of nanoparticles, based on the strain gradient theory. The results showed that the Pasternak shear constant changes the dynamic instability region towards higher speeds.

The above studies have been done on the response of forced vibrations of nanotubes due to a moving load. While due to the existence of nonlinear factors in the equations of motion, complex nonlinear phenomena such as bifurcation, quasi-periodic, and chaotic movements may be created in the dynamic response of the system, which has not been investigated in the above studies [21]. In the following, a few studies conducted on the nonlinear behavior of CNTs are discussed. Zhou et al. [22] used time response diagrams, phase plane, and Poincare sections to detect the chaotic behavior of a CNT placed on a Winkler substrate. Then they introduced threshold values for the occurrence of chaotic behavior in the system. Manani Miandoab [23] investigated the nonlinear behavior of resonators under electrostatic force based on the strain gradient theory. The results showed that the effect of size on the chaotic region in the bifurcation diagram varies significantly depending on the bias voltage. Wang and Zhang [24] studied the chaotic vibrations of a curved CNT in magnetic and thermal fields. The results showed that the parameters of magnetic field intensity and nanotube length can be used to prevent the occurrence of chaotic movements.

The review and summary of previous studies show that the bifurcation analysis of CNTs by simultaneously considering the effect of harmonic moving load, the effect of stiffness of the elastic bed, and the effect of size scale has not been investigated so far. Therefore, in the present study, after presenting a model for CNTs under harmonic moving load based on the theory of non-local elasticity, the equation is discretized with the Galerkin method. Then, the effects of moving load, elastic bed stiffness, and non-local elasticity on the nonlinear dynamic response of the system are investigated by bifurcation diagrams, phase plane, power spectrum, and Poincare sections.

2. Modeling

In Figure 1, the model of a CNT surrounded by an elastic substrate under a harmonic moving force is shown schematically. The CNT has thickness t_b, length L, and modulus of elasticity E. For the system modeling and analysis, the following assumptions are considered:

- The external harmonic force $P(t)$ moves along the neutral axis of the CNT (x-axis) with a constant speed v_p.
- The direction of the external force $P(t)$ is perpendicular to the x-axis.
- The CNT structure is isotropic and homogeneous.
- The boundary conditions at both ends of the nanotube are considered in a simple way.
- The inertial effects of the external harmonic force $P(t)$ have been neglected.
- The initial conditions for transverse vibrations of the nanotube are considered to be zero.

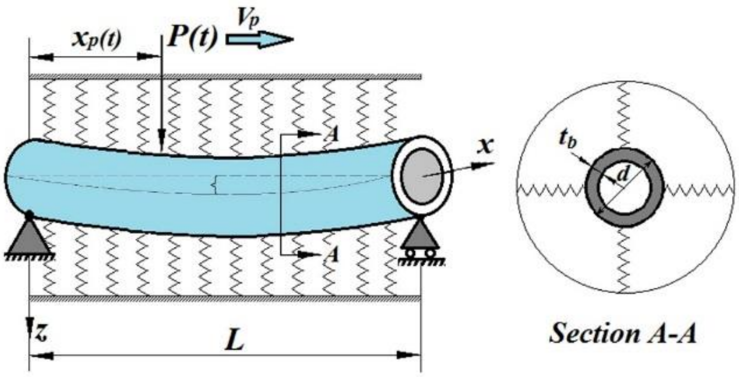


Fig. 1. Carbon nanotube model surrounded by elastic substrate, under moving force.

According to Eringen's theory of non-local elasticity, the nonlocal stress at a particular point is a function of the strain field at all points of the body. The equation of non-local elasticity theory can be expressed as follows [25]:

$$\left(1-\left(e_0a\right)^2\nabla^2\right)\sigma=\tau \tag{1}$$

where e_0, a, ∇, σ, and τ are material constant (determined by experimental methods), internal characteristic length, Laplacian operator, non-local stress tensor, and local stress matrix, respectively. For the Euler-Bernoulli nanobeam, the effects of non-local elasticity in the radial direction can be neglected. Therefore, the non-local stress can be written as follows:

$$\sigma_{xx}-\left(e_0a\right)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\varepsilon_{xx} \tag{2}$$

As can be seen, if the non-local parameter e_0a is considered equal to zero, the theory of classical (local) elasticity is obtained. Considering Euler-Bernoulli beam theory, the governing equation for transverse vibration (along the z-axis) is [26]:

$$-\frac{\partial^2M}{\partial x^2}+p(x,t)=\rho A\frac{\partial^2w(x,t)}{\partial t^2} \tag{3}$$

where $p(x,t)$ is the transverse force distributed on the CNT, ρ is the density of the nanotube, A is the cross-sectional area, $w(x,t)$ is the transverse displacement, t is the time, and M is the bending moment created in the CNT. The bending moment M is expressed as follows:

$$M=-\int z\sigma_{xx}dA \tag{4}$$

where z is the transverse coordinate measured with respect to the x-axis and in the positive direction of the transverse vibrations, additionally, transverse strain ε_{xx} for small deformations is equal to:

$$\varepsilon_{xx} = -z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{5}$$

By combining equations (2), (4), and (5), one can write:

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = EI\frac{\partial^2 w(x,t)}{\partial x^2} \tag{6}$$

where $I = \int z^2 dA$ is the second moment of inertia of the surface. Considering equations (3) and (6), the governing differential equation for the transverse vibrations of the CNT, considering the Euler-Bernoulli beam model and the theory of non-local elasticity, is obtained as follows:

$$\begin{aligned} &EI\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2}{\partial t^2}\left(w(x,t) - (e_0 a)^2 \frac{\partial^2 w(x,t)}{\partial x^2} \right) \\ &= p(x,t) - (e_0 a)^2 \frac{\partial^2 p(x,t)}{\partial x^2} \end{aligned} \tag{7}$$

where m is the mass per unit length of the nanotube. The distributed transverse force $p(x,t)$ is defined as the sum of the external force $F_e(x,t)$ and the force caused by the elastic bed $F_m(x,t)$. Therefore:

$$p(x,t) = F_e(x,t) + F_m(x,t) \tag{8}$$

The moving load causes the external force $F_e(x,t)$. The elastic bed force $F_m(x,t)$ is also described by Winkler and Pasternak's nonlinear model [27]. Thus:

$$F_e(x,t) = P(t).\delta\left(x - x_p\right) \tag{9}$$

$$P(t) = P_0 \sin \Omega t \tag{10}$$

$$F_m(x,t) = -k_1 w(x,t) - k_2 w^3(x,t) \tag{11}$$

where $x_p = v_p.t$ is the position of the moving harmonic load, P_0 is the amplitude of the moving harmonic load, Ω is the frequency of the moving harmonic load, k_1 is the Pasternak linear coefficient, k_2 is the Pasternak nonlinear coefficient, and $\delta(.)$ is the Dirac delta function with the following characteristics:

$$\int_{x_1}^{x_2} g(x).\delta^{(n)}\left(x - x_0\right)dx = \begin{cases} (-1)^n g^{(n)}(x_0) & x_1 < x_0 < x_2 \\ 0 & \text{Otherwise} \end{cases} \tag{12}$$

where $\delta^{(n)}(.)$ is the nth derivative of the Dirac delta function and $g(.)$ is an arbitrary function. Next, the Galerkin method is used to convert the partial differential equation (7) into an ordinary differential equation. Based on this, the vibration response of the system can be estimated as follows:

$$w(x,t)=\varphi(x).q(t) \tag{13}$$

In the above relation, *q(t)* is the generalized coordinate. The first eigenfunction *φ(x)* is defined as follows for the two-sided pin boundary conditions:

$$\varphi(x)=\sin\left(\frac{\pi.x}{L}\right) \tag{14}$$

By placing equation (13) in (7), multiplying the sides of equation (7) by *φ(x)*, and finally integrating in the interval *(0, L)*, the governing equation for the system is obtained:

$$M_0\ddot{q}(t)+Kq(t)+\Lambda q^3(t)=F(t) \tag{15}$$

where

$$M_0=m\int_0^L \varphi^2(x)dx-m\left(e_0a\right)^2\int_0^L \varphi''(x).\varphi(x)dx \tag{16}$$

$$K=EI\int_0^L \varphi''''(x).\varphi(x)dx+k_1\int_0^L \varphi^2(x)dx-k_1\left(e_0a\right)^2\int_0^L \varphi''(x).\varphi(x)dx \tag{17}$$

$$\Lambda=k_2\left(\int_0^L \varphi^4(x)dx-3\left(e_0a\right)^2.\left(2\int_0^L\left(\varphi'(x)\right)^2.\varphi^2(x)dx+\int_0^L \varphi''(x).\varphi^3(x)dx\right)\right) \tag{18}$$

$$F(t)=P(t).\int_0^L\left(\delta\left(x-x_p\right)-\left(e_0a\right)^2\frac{\partial^2\delta\left(x-x_p\right)}{\partial x^2}\right).\varphi(x)dx \tag{19}$$

$$=\begin{cases} P_0\left(1+\left(e_0a\right)^2\left(\pi/L\right)^2\right)\sin\left(\dfrac{\pi v_p t}{L}\right)\sin\Omega t & 0<t<\dfrac{L}{v_p} \\ 0 & \dfrac{L}{v_p}<t \end{cases}$$

In this study, the nondimensional speed parameter α and nondimensional excitation frequency β are introduced as

$$\alpha=\frac{\pi v_p}{\omega L} \tag{20}$$

$$\beta=\frac{\Omega}{\omega} \tag{21}$$

where ω is the natural frequency of the linear system.

3. Results and discussion

A small number of differential equations have an analytical and exact solution, which are often complicated and time-consuming. For this reason, numerical solution methods have become efficient methods for solving differential equations. In this research, the numerical solution of the equation of motion was obtained using the Runge-Kutta method in MATLAB software. The numerical values of physical parameters used for the system modeling are presented in Table 1. The analysis of the nonlinear behavior of the system has been done using phase plane, power spectrum, Poincare sections, and bifurcation diagrams.

Table. 1. Physical parameters of the CNT[10].

Parameter	Value
Young's modulus, E	*1 TPa*
Diameter, d	*1 nm*
Thickness, t_b	*0.35 nm*
Length, L	*40 nm*
Mass density, ρ	*2300 kg/m*^{3}

Phase plane only determines whether the motion of the system is periodic or non-periodic, and cannot provide sufficient information for the occurrence of irregular motion. The power spectrum diagram, which is obtained from the discrete Fourier transform of the output variable, is also used in most signal processing processes. This graph consists of frequency components (spectrum lines), which shows that the target signal can be represented as a discrete set of harmonic functions. If the power spectrum has a spectral line, it indicates periodic movement. The power spectrum of quasi-periodic behavior is also made up of spectral lines at some frequencies that are not proportional to each other. The Poincare section is a plane in the phase space that cuts the phase diagram created by solving the governing equations of the system. For periodic movements, the Poincare cross-section is only one point. If the movement is quasi-periodic, the Poincare section creates a closed curve of discrete points. Bifurcation diagrams are used to identify the path of irregular movement. In the bifurcation diagram, the steady state response of a nonlinear system is drawn as a function of the system's nonlinear parameter. When changing the control parameter, if the movement is subharmonic with the periodicity of n, in each specific step, n component points will be seen on the curve. Finally, if the movement is quasi-periodic or chaotic, a column of points is observed on the curve. In this case, by referring to the Poincare sections, quasi-periodic motion can be distinguished from chaotic motion [28, 29].

3.1. Influence of non-local elasticity parameter

Methods based on continuum mechanics cannot consider the effect of small scales. Therefore, new theories, including the theory of non-local elasticity, have been presented to predict the behavior of nanoscale systems. Figure 2 shows the effect of the non-local elasticity parameter on the bifurcation behavior of the CNT surrounded by the elastic substrate under the harmonic moving force.

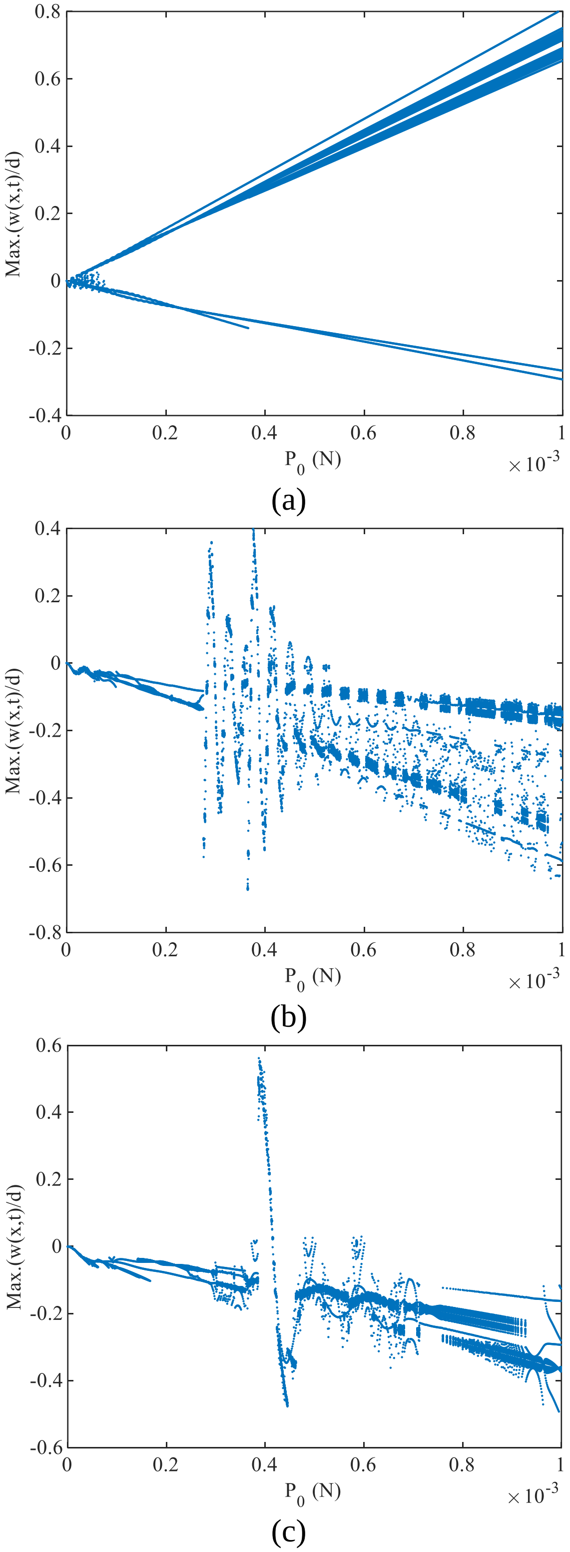
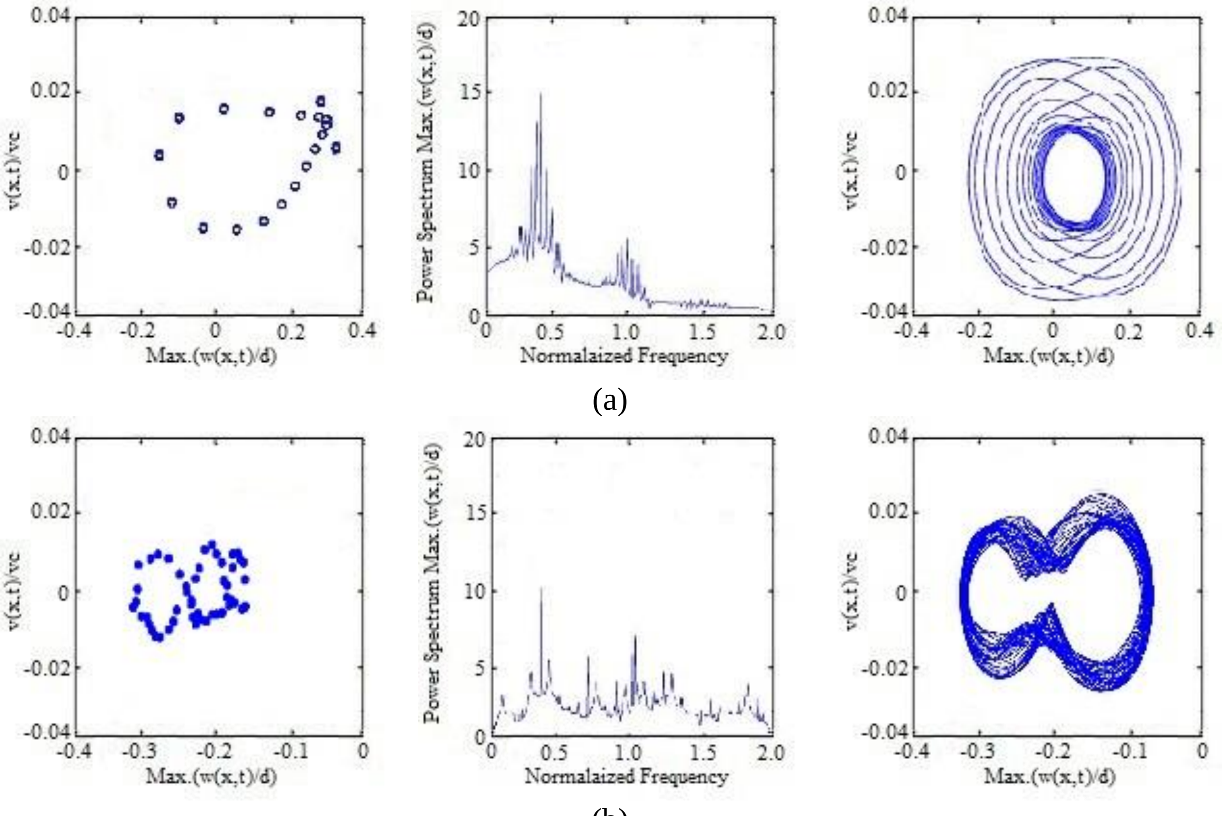


Fig. 2. Bifurcation diagrams of dimensionless displacement of the CNT with the change of excitation force amplitude for $\alpha=0.2$, $\beta=1.0$, $k_1=1\times105\ N/m^2$ and $k_2=4\times1014\ N/m^4$ a) $e_0a=0.0\ nm$ b) $e_0a=0.1\ nm$ c) $e_0a=0.3\ nm$.

For the non-local elasticity parameter $e_0a=0.0$ nm (classical continuum mechanics theory), the nonlinear response of the system remains multi-periodic by increasing the excitation force amplitude to $P_0=0.00025$ N. With the increase of the excitation force, the system starts to behave erratically. Also, by increasing the excitation force, a significant increase in the response of the system is observed.

For the non-local elasticity parameter $e_0a=0.1$ nm, the subharmonic motion is attainable at $P_0=[0.0\text{-}0.00028]$ N. The bifurcation diagram shows irregular motion at a broad range of P_0.

For the non-local elasticity parameter $e_0a=0.3$ nm, the behavior of the system remains subharmonic up to the excitation force $P_0=0.00029$ N. At excitation force $P_0=0.00030$ N, the behavior of the system enters an irregular state and continues until $P_0=0.001$ N. It should be noted that a jump phenomenon also occurs at $P_0=0.00040$ N. Figure 3 shows the phase plane diagram, power spectrum diagram, and Poincare cross section of the response of the CNT in nonlocal elasticity parameter. The excitation force $e_0a=0.0$ nm, $P_0=0.00060$ N, and $e_0a=0.3$ nm, $P_0=0.00080$ N are considered. The frequency components in the power spectrum diagrams and the closed form of the points in the Poincare maps indicate the quasi-periodic behavior of the system for these parameters.



(a)

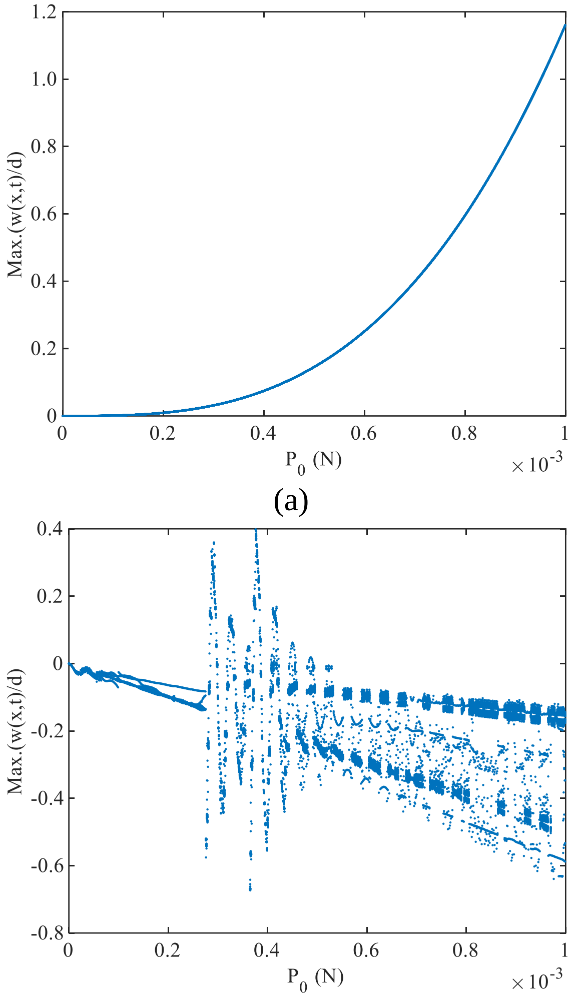
(b)

Fig. 3. Phase diagrams, power spectrum, and Poincare section of dimensionless displacement of the CNT for $\alpha=0.2$, $\beta=1.0$, $k_1=1\times105$ N/m^2 and $k_2=4\times1014$ N/m^4 a) $P_0=0.00060$ N, $e_0a=0.0$ nm b) $P_0=0.00080$ N, $e_0a=0.3$ nm.

By comparing the obtained bifurcation diagrams, it can be concluded that if the effect of size (non-local elasticity parameter) is considered, the occurrence of irregular behavior in the system is more probable. Also, increasing the non-local elasticity parameter (see Figure 2 (b) and 2 (c)) does not transform quasi-periodic movements into chaotic ones for higher excitation force amplitudes. It has a negligible effect on delaying the first irregular behavior.

3.2. Influence elastic bed stiffness

Another parameter affecting the vibration behavior of CNT surrounded by elastic bed under harmonic moving force is the linear and non-linear stiffness of the elastic bed, which are shown in this study by parameters k_1 and k_2, respectively. Figure 4 shows the bifurcation diagram of the CNT response, first without considering the linear stiffness coefficient k_1 and then *with* the nonlinear stiffness coefficient k_2.



(a)

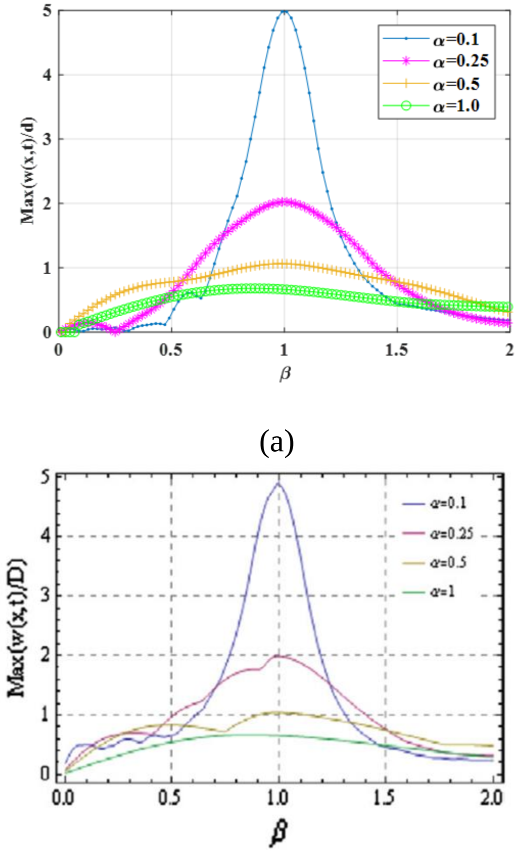
(b)

Fig. 4. Bifurcation diagrams of dimensionless displacement of the CNT with the change of excitation force amplitude per $\alpha=0.2$, $\beta=1.0$, $e_0a=0.1$ nm a) $k_1=0$ N/m^2 and $k_2=4\times1014$ N/m^4 b) $k_1=1\times105$ N/m^2 and $k_2=0$ N/m^4.

Figure 4 (a) shows the bifurcation diagram of the CNT response without considering the linear stiffness of the elastic substrate. Compared to the diagram in Figure 2 (b), in this case, the nonlinear response of the system remains periodic with the increase of the excitation force amplitude. However, with the increase in the range of the excitation force, a significant increase in the response of the system is observed. Figure 4 (b) also shows the bifurcation diagram of the response of the CNT surrounded by the elastic substrate without considering the nonlinear stiffness of the elastic substrate. Compared to the diagram in Figure 2 (b), it can be seen that there is no difference in the type of system response. In other words, the response of the system will not be sensitive to the change of the nonlinear coefficient of the elastic bed.

4. Validation

To verify the proposed computational approach, the variation of the maximum non-dimensional dynamic deflections is compared with results by Hong et al. [11], who neglected the small-size effect and the nonlinear stiffness coefficient. The parameters e_0a and k_2 in equations [[15]–[19]] are set to zero. The maximum non-dimensional dynamic deflections are plotted in Figure 5 and compared with the results presented by Hong et al. [11]



(a)

(b)

Fig. 5. Effects of excitation frequency parameter β on the dynamic deflections of the CNT a) present study b) Ref. [11].

5. Conclusions

In this research, the nonlinear dynamic analysis of CNTs surrounded by a nonlinear elastic bed under a harmonic moving force was investigated. To model the system, Euler-Bernoulli beam theory, non-local elasticity, and Winkler spring have been used. Phase plane paths, power spectrum, Poincaré sections, and bifurcation diagrams were used to analyze the nonlinear behavior of the system. The results showed that the stiffness of the elastic bed and the non-local elasticity parameter have significant effects on the type of dynamic response of the system. Therefore, without the existence of a linear elastic bed, although the response of the system remains periodic, with the increase in the amplitude of the excitation force, the response of the system increases at a significant rate. Moreover, the results showed that by considering the effect of size, the probability of irregular behavior in the system is higher. In addition, the size effect has no considerable effect on delaying the occurrence of the first irregular behavior of the system. Therefore, by using the results of this research and choosing the appropriate parameters, it is possible to prevent irregular behavior in the system.

References

[1] A. Bianco, K. Kostarelos, M. Prato, Applications of carbon nanotubes in drug delivery, Current opinion in chemical biology, Vol. 9, Iss. 6, 674-679 (2005).

[2] R. Chowdhury, S. Adhikari, J. Mitchell, Vibrating carbon nanotube based bio-sensors, Physica E: Low-dimensional Systems and Nanostructures, Vol. 42, Iss. 2, 104-109 (2009).

[3] R. Martel, T. Schmidt, H. Shea, T. Hertel, P. Avouris, Single-and multi-wall carbon nanotube field-effect transistors, Applied physics letters, Vol. 73, Iss. 17, 2447-2449 (1998).

[4] H. Omidvar, M. Sajjadnejad, K. Mirzaei, Z. Sadeghian, S. Shirazi, A. Mozafari, A. Azadmehr, Characterisation of CNTs/TiO2 nano composites and investigation of composite's photo reactivity, International Journal of Materials and Product Technology, Vol. 52, Iss. 3-4, 333-346 (2016).

[5] A.G. Arani, M.S. Zarei, S. Amir, Z.K. Maraghi, Nonlinear nonlocal vibration of embedded DWCNT conveying fluid using shell model, Physica B: Condensed Matter, Vol. 410, 188-196 (2013).

[6] A.G. Arani, S.A. Mortazavi, Z.K. Maraghi, Dynamic stability of nanocomposite viscoelastic cylindrical shells coating with a piezomagnetic layer conveying pulsating fluid flow, Science and Engineering of Composite Materials, Vol. 24, Iss. 3, 401-414 (2017).

[7] A. Ghorbanpour Arani, M. Bagheri, R. Kolahchi, Z. Khoddami Maraghi, Nonlinear vibration and instability of fluid-conveying DWBNNT embedded in a visco-Pasternak medium using modified couple stress theory, Journal of Mechanical Science and Technology, Vol. 27, Iss. 9, 2645-2658 (2013).

[8] A.G. Arani, E. Haghparast, Z.K. Maraghi, S. Amir, Nonlocal vibration and instability analysis of embedded DWCNT conveying fluid under magnetic field with slip conditions consideration, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 229, Iss. 2, 349-363 (2015).

[9] T.-P. Chang, Stochastic FEM on nonlinear vibration of fluid-loaded double-walled carbon nanotubes subjected to a moving load based on nonlocal elasticity theory, Composites Part B: Engineering, Vol. 54, 391-399 (2013).

[10] M. Şimşek, Vibration analysis of a single-walled carbon nanotube under action of a moving harmonic load based on nonlocal elasticity theory, Physica E: Low-dimensional Systems and Nanostructures, Vol. 43, Iss. 1, 182-191 (2010).

[11] Z. Hong, D. Qing-tian, L. Shao-hua, Vibration of a single-walled carbon nanotube embedded in an elastic medium under a moving internal nanoparticle, Applied Mathematical Modelling, Vol. 37, Iss. 10-11, 6940-6951 (2013).

[12] M. Pirmoradian, E. Torkan, D. Toghraie, Study on size-dependent vibration and stability of DWCNTs subjected to moving nanoparticles and embedded on two-parameter foundations, Mechanics of Materials, Vol. 142, 103279 (2020).

[13] H. Sarparast, A. Alibeigloo, V. Borjalilou, O. Koochakianfard, Forced and free vibrational analysis of viscoelastic nanotubes conveying fluid subjected to moving load in hygro-thermo-magnetic environments with surface effects, Archives of Civil and Mechanical Engineering, Vol. 22, Iss. 4, 172 (2022).

[14] J. Natsuki, P. Wu, H. Jiang, T. Natsuki, Dynamic analysis of double-walled carbon nanotubes embedded in elastic medium under a nanoparticle delivery, Diamond and Related Materials, Vol. 128, 109194 (2022).

[15] C. Thongchom, P. Roodgar Saffari, P. Roudgar Saffari, N. Refahati, S. Sirimontree, S. Keawsawasvong, S. Titotto, Dynamic response of fluid-conveying hybrid smart carbon nanotubes considering slip boundary conditions under a moving nanoparticle, Mechanics of Advanced Materials and Structures, Vol. 30, Iss. 11, 2135-2148 (2023).

[16] R. Özmen, I. Esen, Dynamic response of embedded Timoshenko CNTs exposed to magnetic and thermal fields subjected to moving load based on doublet mechanics, Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 45, Iss. 11, 590 (2023).

[17] X. Ma, M. Roshan, K. Kiani, A. Nikkhoo, Dynamic response of an elastic tube-like nanostructure embedded in a vibrating medium and under the action of moving nano-objects, Symmetry, Vol. 15, Iss. 10, 1827 (2023).

[18] J. Leng, T. Chang, Fluid-solid coupling for microscale transport of nanoparticles in ultralong carbon nanotubes, Thin-Walled Structures, Vol. 195, 111431 (2024).

[19] Z.K. Mizuji, M. Ghadiri, A. Rajabpour, M.F. Ahari, A. Zajkani, S. Yazdinia, Numerical modeling of a body vessel for dynamic study of a nano cylindrical shell carrying fluid and a moving nanoparticle, Engineering Analysis with Boundary Elements, Vol. 152, 362-382 (2023).

[20] M. Hashemian, D.J. Jasim, S.M. Sajadi, R. Khanahmadi, M. Pirmoradian, S. Salahshour, Dynamic stability of the euler nanobeam subjected to inertial moving nanoparticles based on the nonlocal strain gradient theory, Heliyon, Vol. 10, Iss. 9, (2024).

[21] H. Ramezannejad Azarboni, S. Edalatpanah, Chaotic vibrations of a harmonically excited carbon nanotube with consideration of thermomagnetic filed and surface effects, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 233, Iss. 10, 3649-3658 (2019).

[22] L. Zhou, F. Chen, Z. Zhao, Subharmonic bifurcation and chaos of a carbon nanotube supported by a Winkler and Pasternak foundation, International Journal of Modern Physics B, Vol. 33, Iss. 19, 1950207 (2019).

[23] E.M. Miandoab, Onset of chaos in nano-resonators based on strain gradient theory: Numerical analysis, Communications in Nonlinear Science and Numerical Simulation, Vol. 101, 105864 (2021).

[24] Q. Wang, Z. Zhang, Chaotic vibration of a curved CNT conveying magnetic fluid in the thermo-magnetic field considering the surface effects, Thin-Walled Structures, Vol. 202, 112047 (2024).

[25] A.C. Eringen, D. Edelen, On nonlocal elasticity, International journal of engineering science, Vol. 10, Iss. 3, 233-248 (1972).

[26] W. Weaver Jr, S.P. Timoshenko, D.H. Young, Vibration problems in engineering, John Wiley & Sons, 1991.

[27] H. Askari, E. Esmailzadeh, Forced vibration of fluid conveying carbon nanotubes considering thermal effect and nonlinear foundations, Composites Part B: Engineering, Vol. 113, 31-43 (2017).

[28] R. Ebrahimi, Chaos in coupled lateral-longitudinal vibration of electrostatically actuated microresonators, Chaos, Solitons & Fractals, Vol. 156, 111828 (2022).

[29] M.M. Jazi, S. Ziaei-Rad, R. Ebrahimi, Chaotic vibration of atomic force microscopes based on the modified couple stress theory, Archive of Applied Mechanics, Vol. 92, Iss. 12, 3683-3694 (2022).