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Detection of localized nonlinearity in dynamical systems using base excitation experimental results

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A B S T R A C T

Nonlinear localization approaches are used not only for detecting the exact location of the nonlinear elements in mechanical structures, but they are also exploited in order to find any possible flaws such as cracks in Structural Health Monitoring (SHM) applications. This study aims to develop a localization method to determine the location of localized nonlinearities in dynamic structures utilizing the experimentally measured data obtained from the base excitation test. The nonlinear element in the experimental set-up is represented by a pair of permanent magnets placed on both sides on the free end of the cantilever, and a pair of electromagnets placed with equal distances on both sides of the permanent magnets. The combination of permanent and electromagnets create and apply nonlinear electromagnetic force on the free end of the cantilever beam. Hence, stepped-sine vibration tests are carried out using constant acceleration base excitation to measure the response of the nonlinear system. The linear response of the system obtained from the low amplitude test is used to update the Finite Element (FE) model of the underlying linear system of the structure. Then, the developed approach utilizes the updated linear model along with the measured nonlinear dynamics of the experimental set-up obtained using high-amplitude excitation to determine the location of nonlinearity. The results of the experimental study are demonstrated to show the performance of the presented method.

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1. Introduction

Almost all engineering structures have nonlinearities coming from various sources such as nonlinear material, geometry, or joints. Of these structures, many can be properly linearized using conventional theories, as their nonlinearities are weak enough to be ignored. However, on the other hand, in many applications, the nonlinearity is stronger than to be neglected. Hence, to have an accurate prediction of the behaviour of the system, the nonlinear model is required to be identified and characterized. There have been numerous studies on the nonlinear identification techniques[1-17], mostly relying on the assumption of the per-known location of nonlinearity. However, in case there is not enough information about the linearity or nonlinearity of the system, a prerequisite for identifying nonlinear elements is to detect the existence of nonlinearities and determine their exact location in engineering structures. Not only for locating the localized nonlinearities, but the localization methods are used also in other applications such as structural health monitoring to detect the location of cracks or unexpected flaws. Accordingly, a multitude of localization approaches has been presented in recent decades[18-24]. Reasonably, all localization methods are based on the comparison between an analytical model of the structure and the experimental data.

In practice, it is not usually possible to measure the response at all coordinates of the structure. For instance, measuring the response of the system in the vicinity of joints is too difficult, or when a relatively huge structure is going to be tested, there may not be enough equipment to measure all required coordinates. Lin and Ewins [20] developed a method based on correlating an analytical model with experimental data to locate the localized nonlinear elements in dynamical structures. The method presented in their study does not require complete measurement at all coordinates, as well as the modelling error is considered in this method. They applied the aforementioned method on both numerical and experimental studies to demonstrate the performance of the method.

Investigating the nonlinear localization approaches based on domain decomposition, Cresta *et al*. [21] proposed two versions of these strategies and applied them to analyse the post-buckling behaviour of long slender structures. They carried out a comparative study on the performances of different methods based on the convergence results. Ondra *et al*. [22] introduced an approach based on the Hilbert transform in the frequency domain and artificial neural networks to detect and identify structural nonlinearities. In this method, training data required for the artificial neural network is created using the frequency response function described by Hilbert transform. To this end, an assumption on the possible types of nonlinearities and corresponding parameter ranges are required. In [23], Koyuncu *et al*. utilized cascaded optimization and neural networks so as to localize and identify the nonlinear elements of dynamical structures. To this end, possible locations of nonlinearities, possible nonlinear forms, and a possible range of parameters' values are selected for the structure considering the physics of the problem. Of course, this requires experience and good engineering sight to avoid missing any possibilities. The assumed possibilities are then used to produced training data using finite element model of the structure. Cascaded optimization and neural networks employ this data set to find, detect, and characterize the nonlinearities of the system. The approach introduced in their study requires an assumption or pre-knowledge about the location and type of nonlinearities, as well as the range of parameter values, which may affect the results of localization and identification of the nonlinear elements.

Developing a localization technique to detect the location of localized nonlinearities based on incomplete measurements, Wang *et al*. [24] tried to eliminate the limitation on the complete measurement for nonlinear localization. The method presented in their study does not require the type of nonlinearity to be assumed or pre-known. Many of the nonlinear localization and identification methods are based on the usage of frequency response of the system. Also, they usually utilize the data obtained from force excited vibrational tests, as it is easier to find the frequency response function in single-input-multi-output tests. The problem here is that applying shaker force through stingers may change the configuration of the systems, particularly for the case of more flexible structures, and this may lead to the considerably different dynamic behaviour of the original structure and make it difficult to identify the system. On the other hand, due to some restrictions, it may be better or more convenient to do the test by base excitation. Therefore, in some cases, it is decided to use base excitation test instead of point force excitation. However, there are two main difficulties in using base excitation vibration test for the methods exploiting frequency response function. The first challenge in this kind of test is that there is no explicit force measurement. In fact, the base motion is transferred to the main structure, and therefore, base excited structural tests are considered as multi-input vibration tests. Accordingly, the second difficulty is to find the required frequency response function in a multi-input-multi-output test. This study aims to develop a localization method to deal with the aforementioned difficulties. This method is developed based on the localization technique introduced by Wang *et al*.[24] to localize the nonlinear elements of a structure using base excited vibrational test data.

Taghipour *et al*. [17] dedicated their paper to the identification and characterization of nonlinear structures using base-excitation vibration test data. In [17], they assumed that the exact location of the nonlinear element of the structure is pre-known. In contrast to their study, the current work is focused on detecting the exact location of the unknown nonlinear elements of the mechanical structures. Besides, in many practical structures, it is not applicable to do vibration test using point force excitation. Therefore, the present study exploits the method introduced by Wang *et al*. [24] to propose a practical approach for localization of structural nonlinearities using the experimental results of base excitation vibration tests.

This paper is focused on the experimental localization of an electromagnetic nonlinear force applied to a base-excited cantilever beam. For this purpose, the base motion of the shaker bed is used as base excitation of the cantilever beam. A combination of two pairs of electromagnets and permanent magnets generates a nonlinear force which is applied to the tip of the cantilever beam. To this end, a symmetric configuration is designed so that permanent magnets are attached to the free end of the cantilever beam, while the pair of electromagnets are placed on the two sides of the permanent magnets. A base motion with a constant amplitude of acceleration is utilized to excite the structure of the cantilever beam. The responses obtained from low-amplitude base motion are exploited to update the underlying linear system. There are many well-developed linear model updating methods in the literature, [25-27], to be used for this purpose. Using the updated model of the underlying linear system along with nonlinear response of the structure resulted by high-amplitude excitation, the exact location of the nonlinear element is detected using the developed localization process based on the method introduced by Wang *et al*.[24].

2. Experiment set-up

The experimental setup considered in this study and its finite element schematic are shown in Fig. 1. The main structure of this setup is a cantilever steel beam. A pair of permanent magnets are attached to the tip of the beam and two electromagnets are located symmetrically on both sides of the permanent magnets, as shown in Fig. 1. At the equilibrium position of the cantilever, symmetric 6-cm gaps exist between two pairs of permanent and electromagnets. Attractive magnetic forces are created in electromagnets by applying a voltage level of 20 V to two electromagnets. These two pairs of permanent magnets and electromagnets generate a nonlinear electromagnetic restoring force which is applied on the tip of the cantilever beam. The cantilever beam is attached to the shaker bed and is excited by the base motion of the shaker with constant acceleration. Stepped-sine excitation method is utilized to measure the nonlinear response of the structure. The displacement of the shaker bed is measured using an accelerometer attached to the shaker, while three other accelerometers are used to measure the translational degrees of freedom 1, 5, and 9, demonstrated in the schematic in Figure 1. These accelerometers have masses of 8 g and attached to the beam as shown in Figure 1. Table 1 contains the geometry and material properties of the experimental setup.

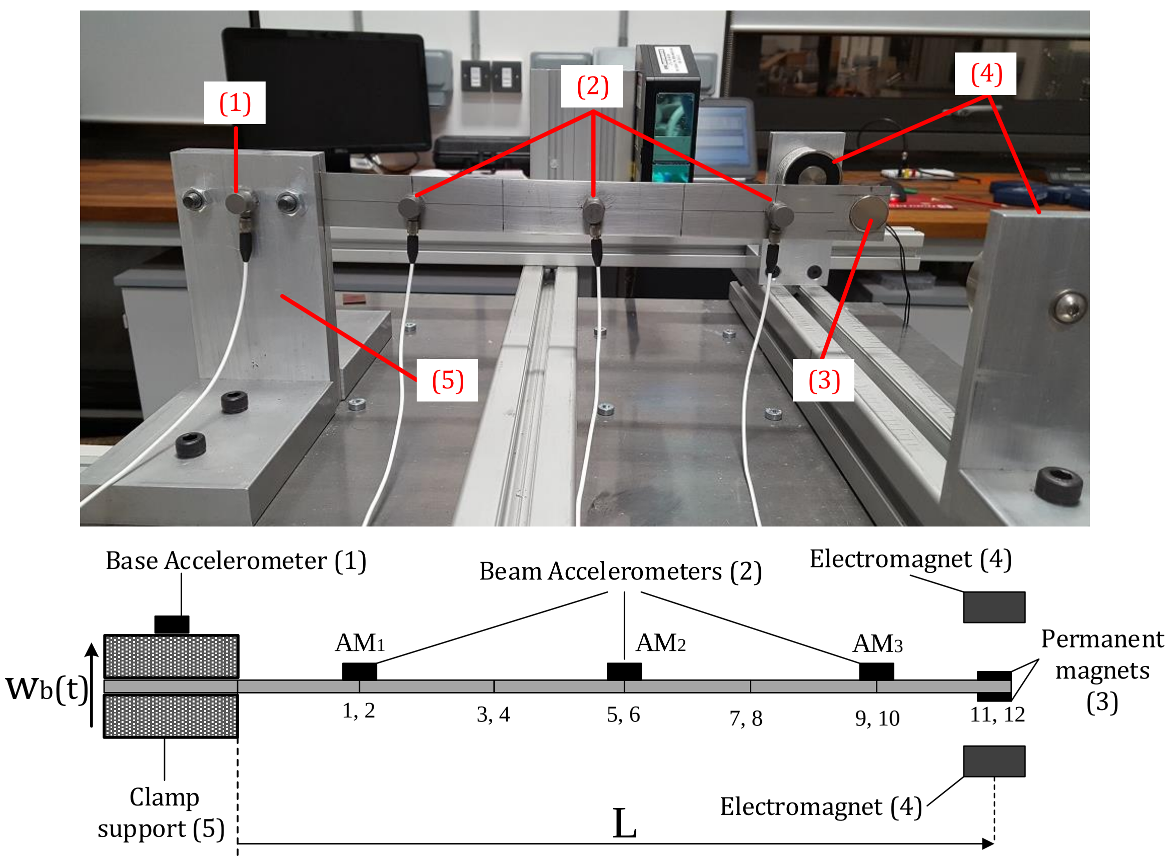

Fig.1: Test-rig assembly and its finite element schematic.

Table 1: Geometry and material properties of the experimental setup.

Parameters	Values
Length of cantilever beam (l)	0.3 m
Width of cantilever beam (d)	0.03 m
Thickness of cantilever beam (t)	0.0015 m
Density of cantilever beam ($\boldsymbol{\rho}$)	7800 $\mathbf{k}\mathrm{g/m^3}$
Modulus of elasticity of cantilever beam (E)	193
Mass of each accelerometer	8 g
Each permanent magnet mass ($\boldsymbol{m_P}$)	4 g
Permanent magnets' pull strength **(N42 Neodymium magnets)**	2 kg
Electromagnets' pull strength	25 kg
Applied voltage to electromagnets (V)	20 V
Initial gap between electromagnets and permanent magnets	6 cm

The following section is devoted to deriving a nonlinear mathematical model for the system of the cantilever beam of Figure 1.

3. Mathematical Model

Considering a cantilever beam with Young's modulus E, density ρ, length L, width d, and thickness h, subjected to base excitation, the finite element model of the structure shown in Fig. 1 can be written as,

$$\widehat{\mathbf{M}}\ddot{\boldsymbol{w}} + \widehat{\mathbf{C}}\dot{\boldsymbol{w}} + \widehat{\mathbf{K}}\boldsymbol{w} + \boldsymbol{f}_{NL}(\boldsymbol{w}, \dot{\boldsymbol{w}}) = \boldsymbol{f}_b(t), \tag{1}$$

where $\widehat{\mathbf{M}}$, $\widehat{\mathbf{C}}$, and $\widehat{\mathbf{K}}$ denote, respectively, the mass, damping, and stiffness matrices of the underlying linear system of the structure, $\boldsymbol{w}$, $\dot{\boldsymbol{w}}$, and $\ddot{\boldsymbol{w}}$ are relative displacement of the beam with respect to the base motion and its time derivatives, respectively, $\boldsymbol{f}_{NL}(\boldsymbol{w}, \dot{\boldsymbol{w}})$ is the nonlinear internal force, and $\boldsymbol{f}_b(t)$ is the equivalent force vector of the base excitation which is applied on the structure. The mass and stiffness matrices of two-node linear Euler-Bernoulli beam elements are obtained as following [28],

$$\mathbf{M}_e = \frac{\rho A L_e}{420}\begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix}, \tag{2}$$

$$\mathbf{K}_e = EIL_e^3\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix}, \tag{3}$$

Calculating the global mass and stiffness matrices using Eqs. (2) and (3), and considering proportional damping for the cantilever beam, the damping matrix can be written as,

$$\hat{\mathbf{C}} = \alpha\hat{\mathbf{M}} + \beta\hat{\mathbf{K}} \tag{4}$$

where α and β are proportional damping coefficients. As the aluminium L-shaped clamp is not completely rigid, it is considered in the model as a cantilever beam, neglecting its torsional motion. Figure 2 demonstrates the equivalent model of the system considering the effect of an L-shaped clamp. The following equations give the expressions for the equivalent mass m_b and linear stiffness k_b of the aluminium clamping beams.

$$m_b = \frac{33}{140}\left(\rho_{Al} V_{Al_b}\right), \qquad k_b = \frac{3E_{Al} I_{Al_b}}{L_{Al_b}^3}, \tag{5}$$

where E_{Al} and ρ_{Al} are respectively Young's modulus and density of aluminium. V_{Al_b}, I_{Al_b}, and L_{Al_b} denote, respectively, the volume, the second moment of area, and the length of the aluminium support beam.

As explained in Section 2, the excitation method used in this study is base excitation. In other words, there is no explicit force vector applied on the structure. In fact, the base movement is transferred to the structure and makes it to oscillate. To find the equivalent force of base excitation applied to the structure, one may utilize the following equation, [27],

$$\boldsymbol{f}_b(t) = -\ddot{z}_b[\hat{\mathbf{M}}]\{\mathbf{g}\}, \tag{6}$$

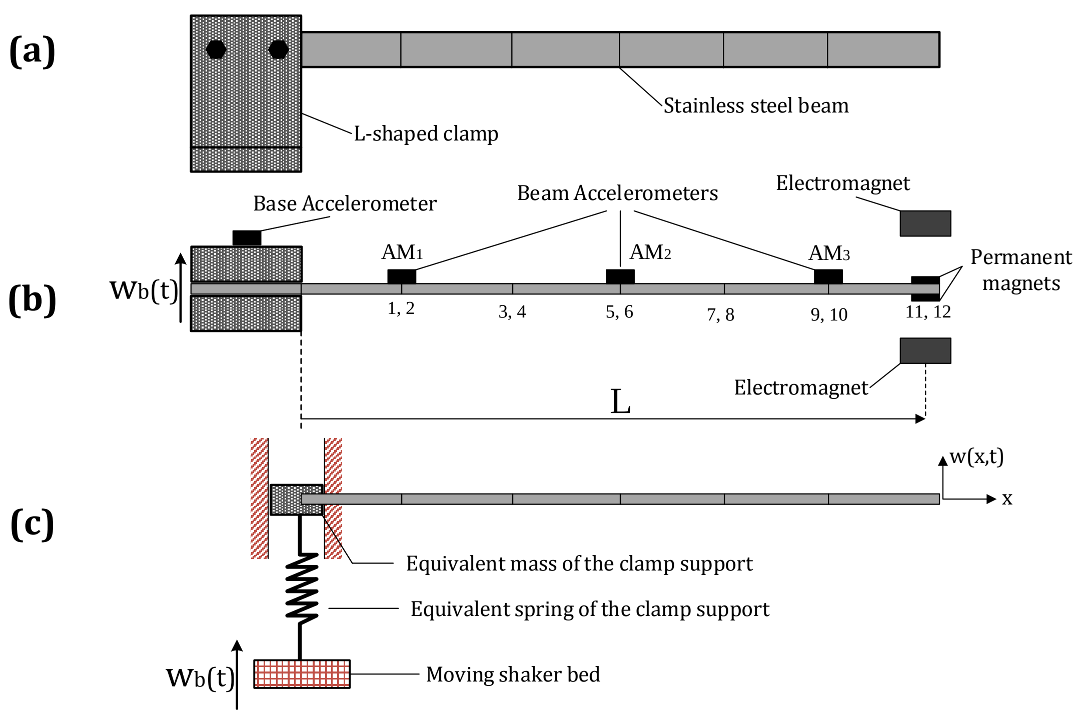

Fig.2: Schematic view of the experimental test-rig including side view (a) and top view (b) of the beam attached to the shaker bed; (c) the equivalent model of the system of shaker bed and the cantilever beam.

where $\boldsymbol{f}_b(t)$ denotes the equivalent force vector of base motion transferred to the structure, M is the mass matrix of the system, $\ddot{z}_b$ is the base acceleration in the time domain, and $\{\mathbf{g}\}$ is the force coordinator vector defined as,

$$g_i = \begin{cases} 1, & \text{if } z_i \text{and } z_b \text{ are in the same direction} \\ 0, & \text{if they are not in the same direction} \end{cases} \tag{7}$$

4. Localization

In this study, the localization process introduced by Wang *et al.*[23] is developed for vibration tests with base excitation. This method is based on the comparison between the responses of the underlying linear system and the nonlinear system. Therefore, two types of vibration tests are required. One is the low amplitude excitation test which is used to update the model of the underlying linear system, and the other one is a high amplitude excitation test giving the nonlinear dynamics of the system required for the localization process.

Hence, the localization process is described below:

a. **Linear model updating**: For both localization and identification of mechanical structures, an accurate model of the underlying linear system is required. Consequently, having any modelling error may lead to inaccuracy in the result of the localization or identification process. Hence, to reduce the modelling error to the lowest possible amount, the measured linear response of the system, obtained from the low-amplitude excitation test, is used to update the preliminary model of the underlying linear system prior to begining the localization process. To this end, there are a variety of linear model updating methods developed in the literature [25-27]. In this study, sensitivity-based linear model updating method presented by Mottershead *et al*. [26] is exploited to update the underlying linear model of the structure. For more details, one may refer to ref. [26]. The updated underlying linear model is obtained as:

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{C}\dot{\mathbf{w}}(t) + \mathbf{K}\mathbf{w}(t) = 0 \tag{8}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are updated mass, damping and stiffness matrices.

b. ***Data selection***: It is not necessary to take into account all the data over the whole frequency span. Indeed, to accelerate the localization process, data selection is carried out to discard the unnecessary data. The localization process is based on the difference between the response of the nonlinear structure and its underlying linear system under the same excitation. Therefore, a high-amplitude excitation signal is applied to the structure to measure the nonlinear response of the system. On the other hand, having the updated underlying linear system, the measured high-amplitude excitation signal is applied to the system of Eq. (8) to simulate the linear response of the system. The deviation $\boldsymbol{\varepsilon}_{nl}$ of the measured nonlinear response from the simulated linear response is defined as:

$$\boldsymbol{\varepsilon}_{nl}(\omega) = \mathbf{w}_{nl}^{ex}(\omega) - \mathbf{w}_{l}^{sim}(\omega) \tag{9}$$

where $\mathbf{w}_{nl}^{ex}(\omega)$ and $\mathbf{w}_{l}^{sim}(\omega)$ are, respectively, the amplitude of measured nonlinear response of the nonlinear structure and simulated linear response of the underlying linear system in frequency domain under the same high-amplitude excitation. Then, a criterion is chosen to select the measured data for the localization process. The criterion is defined so that it guarantees the effect of nonlinearity on the dynamics of the structure to be significant enough

$$\|\boldsymbol{\varepsilon}_{nl}(\omega_i)\|_\infty > \delta_c \tag{10}$$

where $\|\blacksquare\|_\infty$ represents the infinity norm, and δ_c is the threshold for the defined criterion. To minimize the effect of measurement noise and modelling error, δ_c is considered as, [29],

$$\delta_c = \|\mathbf{w}_{nl}^{ex}\| \times (2\sim 5\%) \tag{11}$$

where $\|\blacksquare\|$ denotes the Euclidean norm.

c. ***Nonlinear force assessment***: The method does not require complete spatial measurement at all coordinates. However, the effect of unmeasured degrees of freedom is considered in this method. Accordingly, the coordinates of the system are categorized into measured and unmeasured regions:

$$\mathbf{w} = \begin{Bmatrix} \mathbf{w}_m \\ \mathbf{w}_u \end{Bmatrix}, \tag{12}$$

where the indices m, u represent the measured and unmeasured DOFs, respectively. The measured degrees of freedom are expanded by the SEREP expansion method to predict the response of the system at unmeasured coordinates. Then, the unmeasured DOFs are projected onto the measured region using the Craig-Bampton reduction method. Consequently, through a series of simple mathematics [20], the reduced nonlinear force (RNF) is calculated as a summation of measured nonlinear forces and the projection of unmeasured nonlinear forces onto the measured region.

$$\mathbf{F}_{reduced} = \mathbf{F}_{eq_H} - (\overline{\mathbf{D}}_{mm} - \overline{\mathbf{D}}_{mk}\overline{\mathbf{D}}_{kk}^{-1}\overline{\mathbf{D}}_{km})\mathbf{w}_m, \tag{13}$$

where $\overline{\mathbf{D}}_{mm}, \overline{\mathbf{D}}_{mk}, \overline{\mathbf{D}}_{kk}, \overline{\mathbf{D}}_{km}$ are dynamic stiffness sub-matrices [23]. The index of the reduced nonlinear force is calculated as a summation of the RNFs at the measured DOFs over a range of measured frequencies,

$$\mathbf{I}_{\mathbf{F}^*_{reduced}} = \sum_{\omega_i} |\mathbf{F}^*_{reduced}(\omega_i)|, \tag{14}$$

The suspect region for the location of the nonlinearity is determined using the resulting RNFs, the index of the RNFs, and the phase of the RNFs. Consequently, the preliminary decision for the location of the nonlinear elements is made by minimizing the residual J_{err},

$$\mathbf{F}_{suspect} = \text{ArgMin}\,\|\boldsymbol{J}_{err}\|, \tag{15}$$

Where

$$\boldsymbol{J}_{err} = [\boldsymbol{\Psi}_{um}^T\mathbf{B}_u \quad \mathbf{B}_m]\mathbf{F}_{suspect} - \mathbf{F}_{reduced}, \tag{16}$$

and $\mathbf{B}_u$ and $\mathbf{B}_m$ are the input matrices for the unmeasured and measured DOFs in the suspect region, respectively. Ψ_{um} denotes the matrix of constraint modes in the Craig-Bampton reduction method [27].

Verification: Finally, the result of the location is verified by comparing the nonlinear forces for the suspected region with the reduced nonlinear forces for the measured region

$$\|\boldsymbol{J}_{err}\| \ll \|\mathbf{F}_{reduced}\|, \tag{17}$$

The criterion of Eq. (17) is required to be satisfied with the results of the localization process to be valid.

5. Results and discussion

The section is allocated to the results and discussion. First, a comparison is given between numerically simulated response obtained using the updated linear model of the system and the experimentally measured linear response obtained from the low-amplitude test. Afterward, illustrating the results of the localization process, the application of the localization procedure on the present nonlinear structure is discussed.

As explained, to perform the localization process of the present study, two different sets of vibration tests are carried out; low amplitude and high amplitude tests. For the case of the low amplitude vibration test, the shaker bed is moving with constant acceleration amplitude equal to 2% of the gravitational acceleration g. In order to make sure that the excitation level is sufficient to excite the nonlinearity of the structure, the high-amplitude test with a large enough amplitude of excitation is carried out to measure the nonlinear dynamics of the structure. In this study, the base motion with a constant acceleration amplitude equal to 8% of the gravitational acceleration is used to measure the nonlinear response of the cantilever beam.

In order to minimize the effect of modelling error in the results of localization process, the underlying linear model of the structure is required to be updated using the measured data obtained from a very low-amplitude vibration test. In this study, three parameters of the system (the flexural stiffness EI, base stiffness k_b, and the base mass m_b) are updated using the sensitivity-based updating method [26] and the first three natural frequencies. Then, an experimentally measured damping ratio of the linear response of the system along with sensitivity-based updating method for damping ratio is exploited to update the proportional damping coefficients α and β. Table 2 gives the values of all parameters of the structure considered in updating the underlying linear model.

The updating process has been repeated for 20 iterations. The values of updated parameters EI, k_b and m_b are shown in Figure 3 during the updating process. As illustrated in Figure 4, the updated numerical model is capable of predicting accurately the experimentally measured natural frequencies.

Table 2- Fixed and updated parameters of the underlying linear model.

	Parameters	**Notation**	**Values (units in SI)**	
			Initial guess or fixed values	Updated values
Fixed Parameters	Accelerometers' mass	m_a	0.008	-
	Mass of the permanent magnets	m_p	0.004	-
	Linear mass density of the beam	ρA	0.368	-
	Beam length	L	0.3	-
Updated parameters	Damping coefficient of mass	α	0.2	0.294
	Damping coefficient of stiffness	β	1×10^{-5}	1.33×10^{-5}
	Flexural stiffness of the beam	EI	1.741	1.64
	Equivalent stiffness of the clamp support	k_b	8.256×10^5	5.5×10^5
	Equivalent mass of the clamp support	m_b	0.1016	0.103

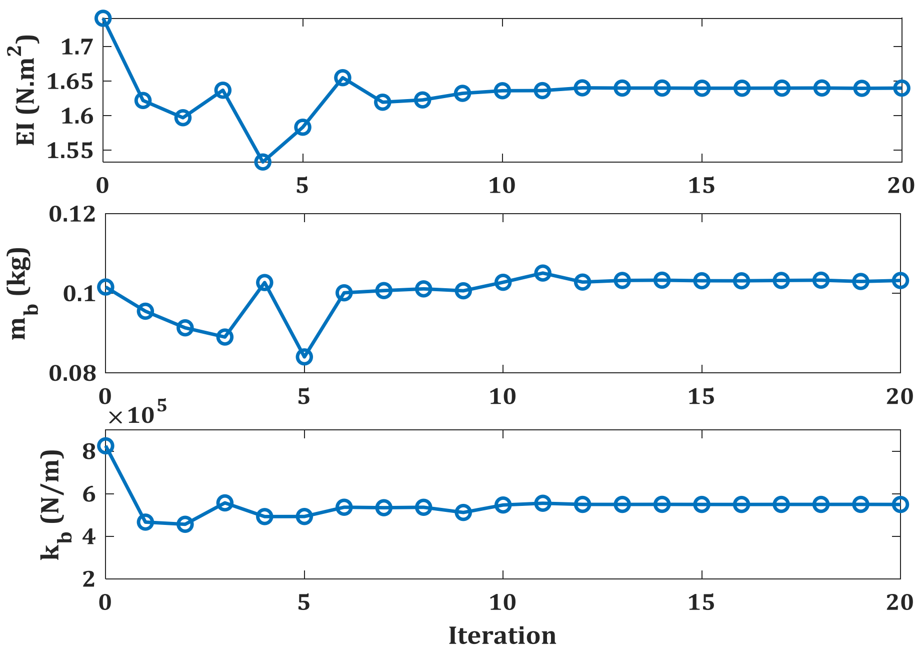

Fig.3: The parameters updated in the process of linear model updating.

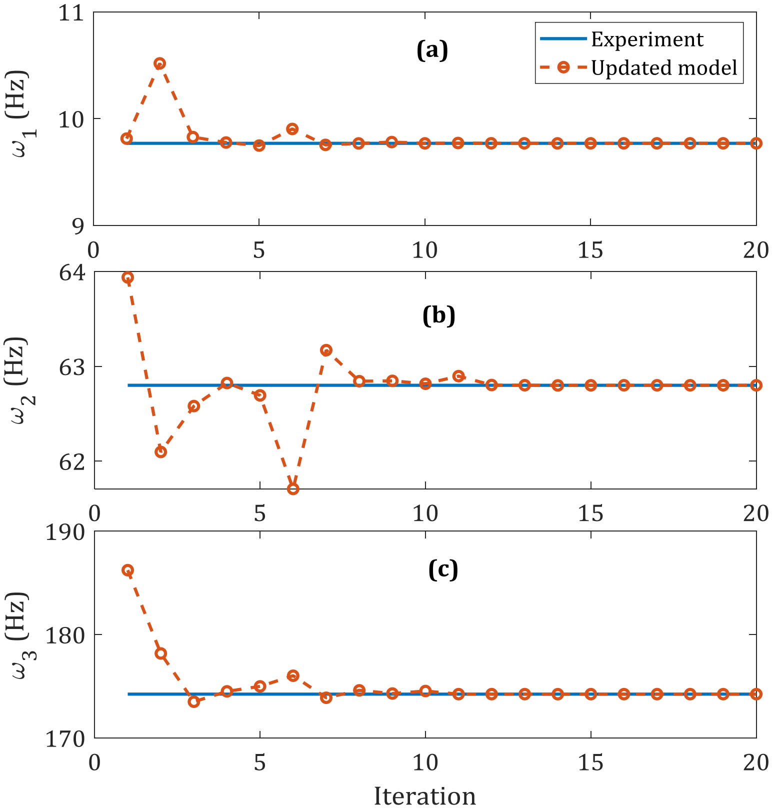

Fig. 4: The numerical natural frequencies obtained using the updated linear model versus the experimentally measured ones.

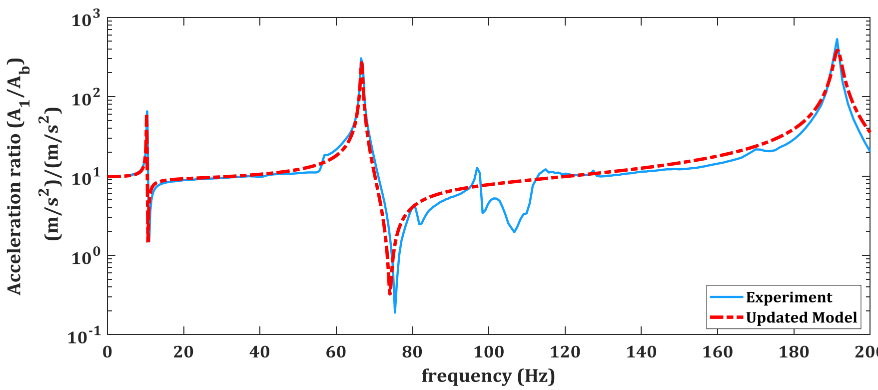

Fig. 5: The experimentally measured frequency response compared with the updated numerical frequency response of the cantilever beam at DOF1. This figure shows the ratio of the amplitudes of accelerations as the frequency response of the structure.

The numerical linear frequency response of the structure is simulated exploiting the updated parameters of the underlying linear model. Figure5 illustrates the numerically simulated frequency response in comparison with the measured dynamics of the structure at DOF1 within the frequency range of $(0 - 200)$ Hz including the first three natural frequencies. It is observed that the anti-resonance frequencies can be accurately predicted by the updated model, as well as the resonance frequencies. Here it should be mentioned that the updating process does not utilize the anti-resonance frequencies. As a result, the numerically updated results being in excellent accordance with experimentally measured both resonance and anti-resonance frequencies verifies the validity of the updated linear model. Figure 6 gives a comparison between the low-amplitude experimental and updated numerical linear responses at the vicinity of the first resonance, taking the effect of nonlinear electromagnetic force into consideration both in experimental and numerical results.

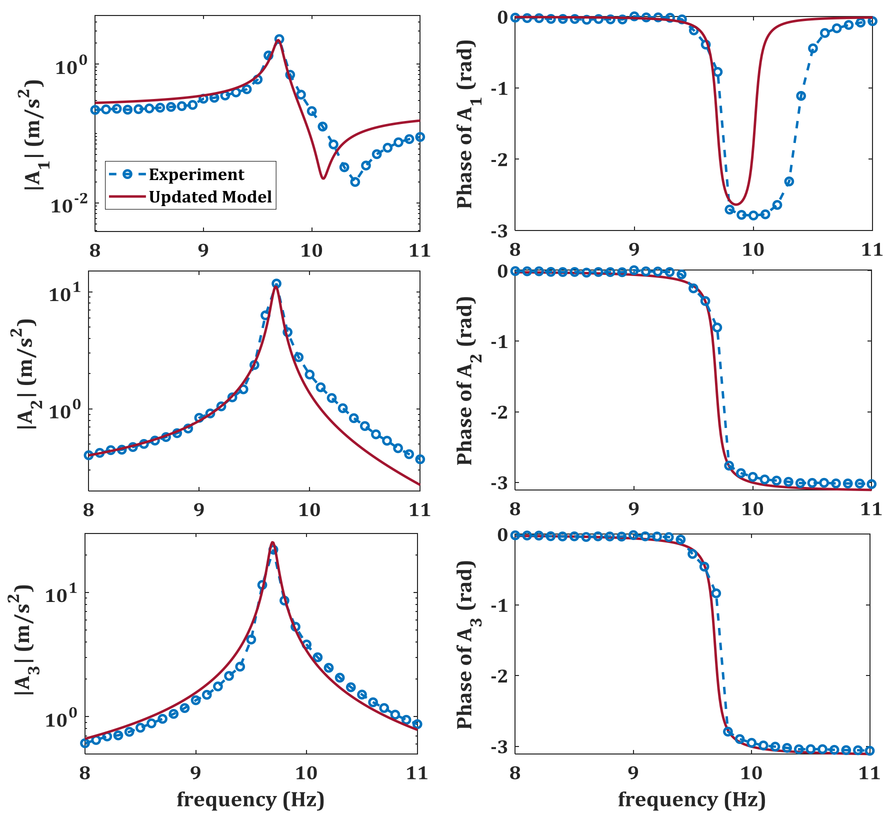

Fig. 6: The updated linear response in comparison with the measured dynamics of the nonlinear structure obtained from the low amplitude vibration test. $\boldsymbol{A_i}$ $(\boldsymbol{i = 1, 2, 3})$ denotes the amplitude of the accelerations measured by three accelerometers, respectively.

After updating the linear finite element model, this linear model is used to detect the exact location of the nonlinear element of the structure. Utilizing the updated linear model and the experimental data from the high amplitude tests, the reduced nonlinear forces of the system are obtained according to the localization process. To this end, the structure is subjected to a constant acceleration base excitation with a large amplitude of $0.08\,g$. The high-amplitude constant acceleration of the movement of the shaker bed is shown in Figure 7 for the frequency span of 8.9 Hz to 11 Hz. It is worth mentioning that the interaction between the shaker and the dynamics of the system at the vicinity of the natural frequency may lead to variation in the excitation level. Hence, a closed-loop (controlled) stepped-sine vibration test is performed in order to control the excitation signal. Forward and backward sweeps may be required to obtain all stable solutions of the system, particularly in the region of multi-solution response. However, taking the upper branch of the nonlinear response (with larger amplitude) into consideration guarantees the nonlinearity of the dynamic response to be considered in the calculation. This is because, the bigger difference between nonlinear and linear responses, the better the results of the localization process. On the other hand, the system of the present study has softening nonlinearity. Therefore, only the backward sweep (sweep-down) test, which provides the upper branch of the response, is sufficient. The amplitude of the excitation is selected as large as it makes sure the system shows a nonlinear dynamic response. Figure 8 illustrates the response-frequency diagram of the nonlinear dynamics of the system, including amplitudes and phases at three DOFs 1, 5, and 9 which are related to the translational displacement of the points with distances 5, 15, and 25 cm from the fixed end of the cantilever beam, respectively. The localization process is carried out using the measured nonlinear responses of the structure for the purpose of detecting the exact location of the unknown nonlinear restoring force.

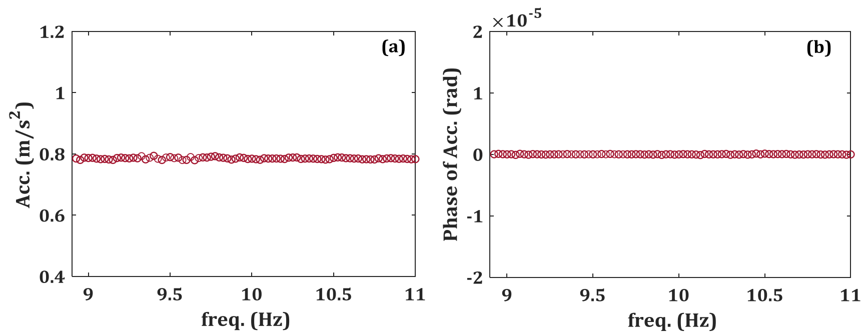

Fig. 7: Controlled constant acceleration base motion; (a) amplitude, (b) phase (b).

The localization is carried out using the measured nonlinear responses, according to the process explained in Section 4. Figure 9 illustrates the magnitude and phase of the reduced nonlinear force at measured DOFs. The indices of the reduced nonlinear force for measured DOFs 1, 5, and 9 are shown in Figure 10, demonstrating that the nonlinear element is located at or close to DOF9. On the other hand, Figure 9 illustrates a phase difference of 180 degrees between DOFs 5 and 9, while the index of the nonlinear force of DOF9 is shown in Figure 10 to be much greater than the one of DOF5. Therefore, it can be concluded that there is no ungrounded (connected) nonlinear element between DOFs 5 and 9. Conclusively, DOFs 1 and 5 can be discarded from the suspect region of nonlinearity. Besides, DOF3 located between DOFs 1 and 5 can be discarded from the suspect region. As a result, DOFs 7, 9, and 11 are considered as suspect DOFs. The nonlinear force has been calculated for the suspect region. Looking at the indices of nonlinear forces in the suspect region, Figure 11, one can interpret that the nonlinear element may be at both DOFs 9 and 11. However, looking at the difference between indices of nonlinear forces of two DOFs, the nonlinear force can be considered as a localized grounded nonlinearity at DOF 11. Using the results of the localization process, the type of nonlinearity can be identified via different identification methods.

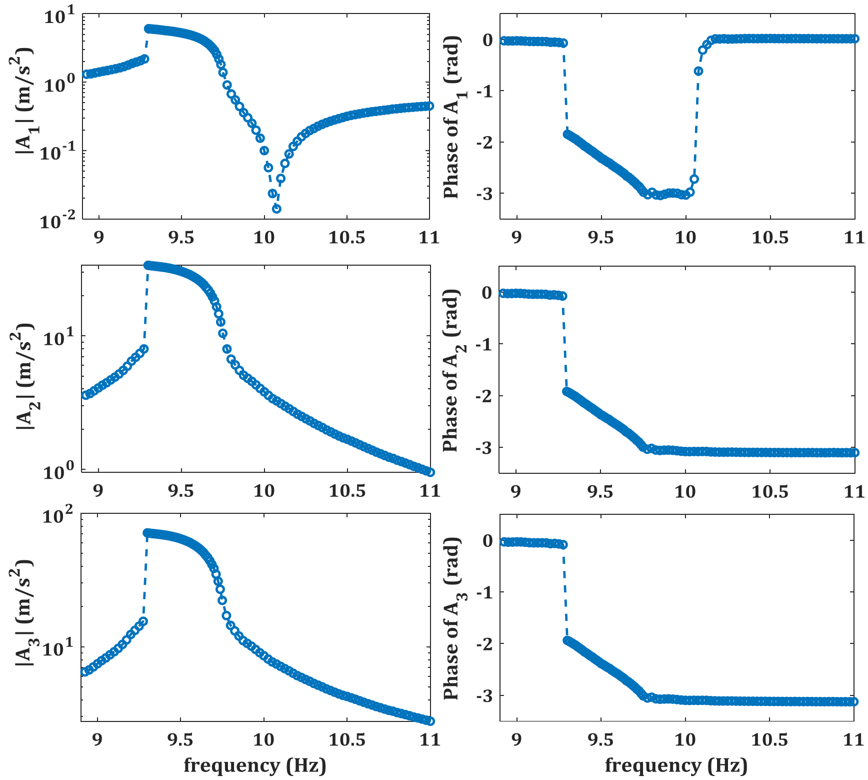

Fig. 8: The measured nonlinear dynamics of the system captured using a high amplitude vibration test. $|A_1|$, $|A_2|$, and $|A_3|$ represent the amplitude of the accelerations respectively measured by three accelerometers.

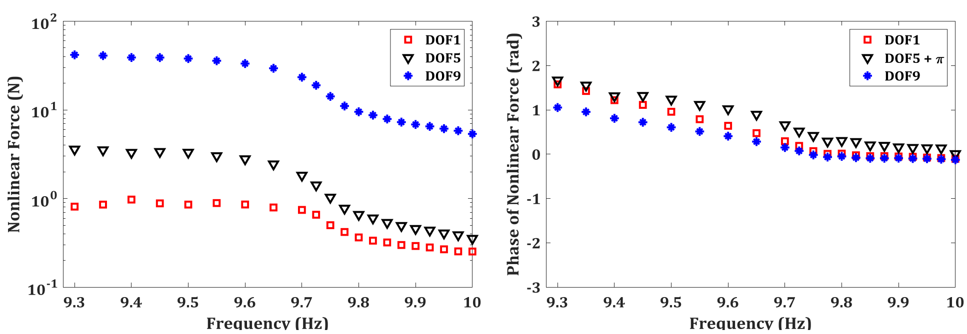

Fig. 9: Magnitude and phase of the reduced nonlinear force of the measured degrees of freedom.

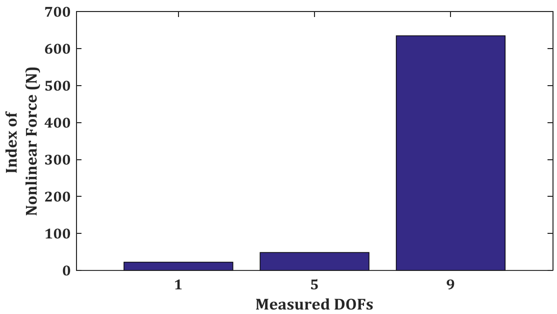

Fig. 10: Indices of the nonlinear forces at the measured DOFs

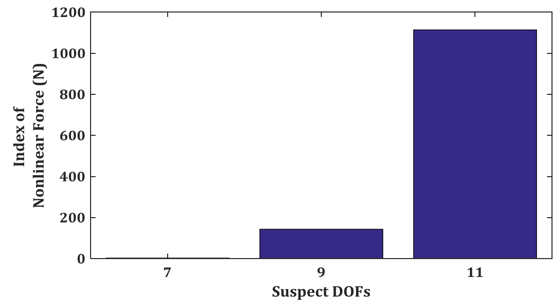

Fig. 11: Indices of the nonlinear forces at the suspect DOFs.

6.Conclusion

This study has aimed to detect the location of localized nonlinearity in structural dynamics using base excitation experiments. To this end, a localization method has been employed to localize an electromagnetic nonlinear force applied on the tip of a cantilever. A symmetric configuration of two pairs of electromagnets and permanent magnets generates a nonlinear force which is applied to the tip of the cantilever beam. For the purpose of localization, two types of vibration tests have been carried out. Low-amplitude test results were used to update the model of the underlying linear system of the structure, while the results of the high-amplitude test are required for the localization process itself. The results of the localization demonstrate the capability of the method utilized to detect the location of nonlinear elements in the structure under study.

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