I S A V

Journal of Theoretical and Applied

Vibration and Acoustics

journal homepage: http://tava.isav.ir

Nonlinear Dynamics and Control of Crank-Slider Mechanism with Multiple Clearance Joints

Sadeq Yaqubi, Morteza Dardel *, Hamidreza Mohammadi Daniali, Mohammad Hassan Ghasemi

Department of Mechanical Engineering, Babol Noshirvani University of Technology, Postal Code: 47148-71167, Shariati Street, Babol, Mazandaran, Iran

K E Y W O R D S

Multiple clearance joints

Joint friction

Nonlinear dynamics control

A B S T R A C T

In the current study, behavior of crank-slider mechanism with single and multiple clearance joints are analyzed. Using Lankarani-Nikravesh theory for estimating discontinuous contact forces in clearance joints, relevant systems have been mathematically modeled. Through numerical simulations, perturbations in response of mechanisms with clearance joints have been analyzed. Effects of increasing number of clearance joints have been addressed. From comparisons between responses of crank-slider mechanism with a single clearance joint and multiple clearance joints, it is concluded that perturbations intensify as the number of clearance joints in mechanism increases. Nonlinear dynamics of system are analyzed, using Poincare maps and bifurcation diagrams. Effects of joint friction on the response of the mechanism are investigated. Subsequently, a control scheme for providing continuous contact in clearance joints and maintaining a more stable mechanism is pro-posed. Obtained results demonstrate the effectiveness of proposed control method on reducing effect's of clearance and maintaining continuous contact in clearance joint.

1. Introduction

Because of the increasing need for higher operational precision, the importance of achieving higher accuracy in analysis and control of mechanical systems is becoming more apparent day by day. This issue demands construction of more complete models, considering physical and dynamical effects that have been normally ignored. Imperfection of joints is among such features; i.e., considering joints as actual physical apparatus with clearance that notably affect system's response notably improves analytical accuracy. As in Earls and Wu[1], many of studies regarding clearance in mechanisms assume continuous contact in joints. In this approach, clearance is simply modeled as massless link with known length connecting journal center to bearing. While not very accurate and relatively outdated, massless link model is not defunct yet however, and has been applied in more recent studies such as investigations by Erkayaa and Uzmay (2012) [2] for comparing theoretical analysis to experimental set-up in the case of crank-slider mechanism with clear-acne and by Dupac and Beale (2010) [3] for analyzing the effects of cracks in flexible linkages. The investigations of Dubowsky et al. [4] can be cited as an important breakthrough regarding the study of clearance joints. They proposed a model in order to estimate contact forces in joints caused by elasticity and energy loss. Subsequent theories proposed more sophisticated and accurate contact models, which were usually built upon Hertz' elastic theory [5], though Hertz' theory does not include the important effect of

*Corresponding Author: Morteza Dardel, Email: dardel@nit.ac.ir

energy dissipation during contact and thus cannot be used on its own to investigate dynamical effects of clearance joints. Various models have been proposed for addressing the problem of energy dissipation, leading to more accurate estimations of contact force, the most famous of which is Lankarani-Nikravesh model (1990) [6]. In current study, this model is used for calculating contact force. In Lankarani-Nikravesh model, three modes of joint configuration are assumed; i.e., (a) free flight motion: in which journal and bearing have no contact, (b) impact mode: the occurrence of impact between the journal and the bearing; at this moment free flight motion ends and continuous contact is established afterward, and (c) continuous contact: in which the journal and the bearing are in contact and as the result of relative penetration, contact force is exerted on bodies. Beside Lankarani-Nikravesh models, other contact theories have also been proposed for analysis of mechanisms with clearance, the effectiveness of which has been studied by Koshy et al. [7].

As a result of complicated motion processes in systems with joint clearance and transfer of nonlinear forces and torques, these systems frequently exhibit nonlinear and chaotic dynamics [8]. Moreover, joint friction, which is normally ignored in the dynamical analysis, is another important parameter in clearance joints. Columba friction models have usually been used in literature for to discuss joint friction in mechanisms with clearance joints. However, including the effects of joint friction will cause additional complexities in numerical simulations [9] and therefore it has been frequently overlooked. To address this issue, researchers such as Ambrosia (2002) [9] have suggested considering additional friction coefficient covering several situations. Olyaei and Ghazavi (2012) [10] have incorporated Ambrosia's friction into the contact force model proposed by Lanka-rani-Nikravesh though they also have ignored it in numerical simulations. It can be seen that joint friction is a topic much more frequently discussed than actually numerically analyzed. Moreover, the majority of studies have focused on the behavior of mechanisms with single clearance joint and effects of multiple clearance joints has been mostly overlooked, though the investigations of Flores [11] and Cheng [12] can be cited as notable exceptions. Moreover, unwanted effects of mechanisms with joint clearances such as shortened lifetime, notable extra noise and lower positional accuracy can be prevented or reduced by methods such as appropriate design or by preventing contact loss in clearance joints [13]. More recently Olyaei and Ghazavi (2012) [10] used delayed feedback control to reduce perturbations of crank-slider mechanism.

This study attempts to achieve further understanding of behavior of mechanisms with clear-acne joints and reduce the undesired effects of joint clearances. Responses of crank-slider mechanism with multiple clearance joints are analyzed. Nonlinear dynamical behavior of system is stud-ide using Poincare maps and bifurcation diagrams. Effects of additional clearance joints are studied through comparisons to mechanisms with a single clearance joint and mechanism with perfect joints. Joint friction, which is usually ignored in studies regarding joint clearances because of the various complexities it proposes, is investigated. Eventually a control scheme with aim of maintaining continuous contact has been proposed and successfully applied.

2. Derivation of equation of motion

2.1. Contact force modeling

According to Lankarani and Nikravesh [6] model, considering hysteresis damping in addition to elasticity, normal contact force in clearance joint is calculated as:

$$F_N = K\delta^n\left[1+\frac{3(1-c_e^2)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{1}$$

K represents general stiffness and δ is the relative penetration depth. For the movement of the journal within the bearing, the penetration depth is equal to:

$$\delta = r-\left(R_B-\ R_J\right) \tag{2}$$

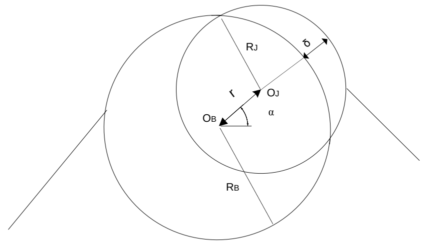

Fig 1: Schematic of revolute clearance joint

r is the distance between the journal centre and the bearing centre. R_B and R_J , respectively, represent radius of the bearing and radius of journal. If tension distribution is circular or elliptical, exponential factor n is set to 1.5. Contact force is exerted to the system during continuous contact mode. Free flight mode can be described as the state in which $\delta < 0$ and continuous contact mode can be described as the state in which $\delta > 0$. The moment this transition takes place, an impact occurs. Because of tangential component of relative motion, joint friction should be considered. A modification of Columba friction is usually used; however this model results in discontinuities at near zero tangential velocities [9]. Subsequent modifications such as Ambrosia model (2002) [9] have bypassed this problem. In Ambrosia's model, an additional coefficient is considered whose value depends on tangential velocity of motion.

$$F_T = -C_d C_f F_N \frac{v_T}{|v_t|} \tag{3}$$

$$C_d = \begin{cases} 0 & |v_t| > v0 \\ \dfrac{|v_t| - v_0}{v_1 - v_0} & v_0 \leq |v_t| \leq v_1 \\ 1 & |v_t| > v0 \end{cases} \tag{4}$$

In which C_f is the coefficient of friction. Moreover, C_d represents dynamic correction coefficient. v_0 and v_1 are relative tangential velocity tolerances. Total contact force is calculated by adding tangential element to normal component of contact force, the magnitude Q_c and its angle ψ have been described by Olyaei et al. [10] as:

$$Q_c = K_{eq}\delta^n \left[1 + \frac{3(1-c_e^2)}{4}\frac{\dot{\delta}}{\dot{\delta}(-)}\right] \tag{5}$$

$$\psi = \alpha + \varphi \tag{6}$$

where:

$$K_{eq} = \sqrt{1 + C_f^2 C_d^2}\, K \tag{7}$$

$$\varphi = \tan^{-1}\left(C_f C_d \frac{r\dot{\alpha} + R_J\omega_J - R_B\omega_B}{r\dot{\alpha} + R_J\omega_J - R_B\omega_B}\right) \tag{8}$$

ω_B and ω_J denote angular velocities of the journal and the journal, and α is the clearance angle depicted in Figure (1). It is observed that friction causes an increase in general stiffness of the system and changes the orientation of contact forces.

2.2. Equations of motion

2.2.1. Equation of motion for mechanism with single clearance joint

In case of mechanism with single joint clearance, clearance joint is placed between the slider and connecting rod. As it is depicted in Figure (2), state parameters in this case are chosen as the angle of the crank (θ_2), angle of connecting rod (θ_2), and horizontal position of the slider (x). The origin is placed at the joint between the crank and the base.

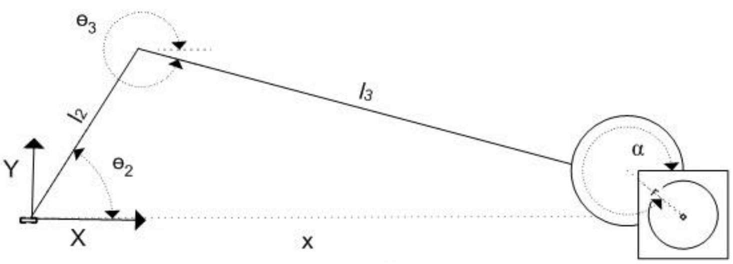

Fig 2: Crank- slider mechanism with single clearance joint.

Governing equations of motion are derived as:

$$m_4\ddot{x} = -Q_c\, cos\,\psi \tag{9}$$

$$\begin{gathered}(I_3 + m_3a_3^2l_3^2)\,\ddot{\theta}_3 + m_3a_3l_2l_3\cos(\theta_2 - \theta_3)\,\ddot{\theta}_2 - m_3a_3l_2l_3\sin(\theta_2 - \theta_3)\,\dot{\theta}_2^2 \\ + m_3gl_3a_3\cos\theta_3 = -Q_cl_3\sin(\theta_3 - \psi)\end{gathered} \tag{10}$$

$$\begin{aligned}\big((m_2a_2^2 + m_3)l_2^2 + I_2\big)\ddot{\theta}_2 &+ m_3a_3l_3l_2\ddot{\theta}_3\cos(\theta_2 - \theta_3) \\ &+ m_3a_3l_3l_2\dot{\theta}_3^2\sin(\theta_2 - \theta_3) + (m_3 + m_2a_2)gl_2\cos\theta_2 \\ &= M - Q_cl_2\sin(\theta_2 - \psi)\end{aligned} \tag{11}$$

In formulations above, m_i , I_i and l_i respectively describe the mass, moment of inertia and length of the link i according to figure 2, m_4 represents the mass of slider. a_il_i describes the distance between centre of mass of link i and the joint connecting it to link $i - 1$. Clearance radius and clearance angle are obtained from geometric relations.

Clearance radius and clearance angle are obtained from geometric relations.

2.2.2. Equation of motion for mechanism with multiple clearance joints

It is considered that an additional clearance joint exists between crank and connecting rod. Compared to mechanism with single clearance, two extra parameters are needed to describe system, which are chosen as horizontal and vertical position of the connecting rod's mass centre (X_{G_3}, Y_{G_3}). The mechanism has been schematically depicted in Figure (3).

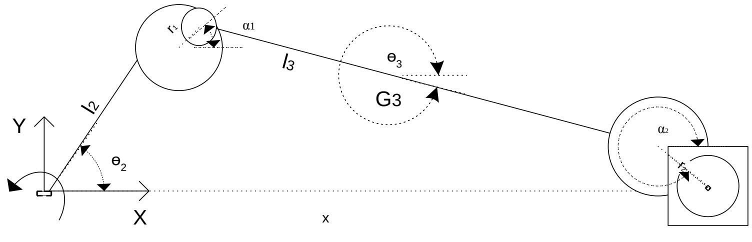

Fig 3: Crank -slider mechanism with multiple clearance joints.

In formulations below, subscript "1" refers to the clearance joint between crank and connecting rod (Q_{c_1}, r_1) and subscript "2" refers to clearance joint between the connecting rod and the slider (Q_{c_2}, r_2). Equations of motion are obtained as:

$$(m_2 a_2^2 l_2^2 + I_2)\,\ddot{\theta}_2 + mgl_2 a_2 \cos\theta_2 = M - Q_{c_1} l_2 \sin(\theta_2 - \psi_1) \tag{12}$$

$$I_3\ddot{\theta}_3 = -Q_{c_1}\frac{l_3}{2}\sin(\theta_3 - \psi_1) - Q_{c_2}\frac{l_3}{2}\sin(\theta_3 - \psi_2) \tag{13}$$

$$m_3\ddot{X}_{G_3} = -Q_{c_1}\cos\psi_1 + Q_{c_2}\cos\psi_2 \tag{14}$$

$$m_3\ddot{Y}_{G_3} + mg = -Q_{c_1}\sin\psi_1 + Q_{c_2}\sin\psi_2 \tag{15}$$

$$m_4\ddot{x} = -Q_{c_2}\cos\psi_2 \tag{16}$$

Clearances radii and angles are obtained through geometric relations.

2.3. Control procedure

Preventing contact loss in crank-slider mechanism with clearance joint will result in avoiding discontinuous forces, reducing fatigue and maintaining more stable dynamical behaviors along with considerably higher positional accuracy. To this end, continuous contact is considered as a system constraint. The mechanism starts from a condition in which the journals and the bearings are in contact and under this constraint, this condition will be maintained during the motion. For main-training continuous contact at joints, the following constraint is considered.

$$g_i = r_i^2 - \left(R_{B_i} - R_{J_i}\right)^2 = 0 \tag{17}$$

2.3.1. Control of mechanism with a single clearance joint

Since links are in continuous contact, contact force acts as an internal force between the bod-ies, therefore the newfound value of contact force can be obtained using appropriate dynamical rela-tions. From equation (9), we now obtain:

$$Q_c = -\frac{m_4\ddot{x}}{\cos\alpha} \tag{18}$$

Updating equations (10-11) with the newfound formulation of Q_c obtained from equation (9), along with using the constraint equation (17), the governing equations for the crank-slider mechanism with continuous contact between the journal and the bearing can be derived for the mechanisms with a single clearance joint.

$$\begin{aligned}\left((m_2a_2^2 + m_3)l_2^2 + I_2\right)\ddot{\theta}_2 + m_3a_3l_2l_3\cos(\theta_2-\theta_3)\,\ddot{\theta}_3 - m_4l_2\frac{\sin(\theta_2-\alpha)}{\cos\alpha}\ddot{x}\\ + m_3a_3l_3l_2\dot{\theta}_3^2\sin(\theta_2-\theta_3) + (m_3 + m_2a_2)gl_2\cos\theta_2 = M\end{aligned} \tag{19}$$

$$\begin{aligned}m_3a_3l_2l_3\,cos\,(\theta_2-\theta_3)\,\ddot{\theta}_2 + (I_3 + m_3a_3^2l_3^2)\ddot{\theta}_3 - m_4l_3\,sin\,(\theta_3-\alpha)\,\ddot{x}\\ - m_3a_3l_3l_2\dot{\theta}_2^2\,sin\,(\theta_2-\theta_3) + m_3gl_3a_3\,cos\,\theta_3 = 0\end{aligned} \tag{20}$$

$$\begin{aligned}&[2xl_2\sin\theta_2 - 2l_2l_3\sin(\theta_2-\theta_3)]\ddot{\theta}_2 + [2xl_3\sin\theta_3 + 2l_2l_3\sin(\theta_2-\theta_3)]\,\ddot{\theta}_3\\ &+ [2x - 2l_2\cos\theta_2 - 2l_3\cos\theta_3] + 2\dot{x}^2 + 4\dot{x}\dot{\theta}_2l_2\sin\theta_2 + 2xl_2\dot{\theta}_2^2\cos\theta_2\\ &+ 4\dot{x}\dot{\theta}_3l_3\sin\theta_3 + 2x\,l_3\dot{\theta}_3^2\cos\theta_3 - 2l_2l_3\left(\dot{\theta}_2-\dot{\theta}_3\right)^2\cos(\theta_2-\theta_3) = 0\end{aligned} \tag{21}$$

Subsequently, necessary input torque for establishing this dynamical behaviour can then be calculated according to the formulations above.

2.3.2. Control of mechanism with multiple clearance joint

In this case, constraints of continuous contacts are written as:

$$\left(X_{G_3}-l_2\cos\theta_2-l_3a_3\cos\theta_3\right)^2+\left(Y_{G_3}-l_2\sin\theta_2-l_3a_3\sin\theta_3\right)^2-\left(R_{B_1}-\ R_{J_1}\right)^2=0 \tag{22}$$

$$\left(x-X_{G_3}-\ l_3(1-a_3)\cos\theta_3\right)^2+\left(-Y_{G_3}-\ l_3\ (1-a_3)\sin\theta_3\right)^2-\left(R_{B_2}-\ R_{J_2}\right)^2=0 \tag{23}$$

Then, contact forces are obtained from (14) and (16).

$$Q_{c_1}=\ \frac{-m_3\ddot{X}_{G_3}-\ m_4\ddot{x}}{\cos\alpha_1} \tag{24}$$

$$Q_{c_2}=\ -\frac{m_4\ddot{x}}{\cos\alpha_2} \tag{25}$$

Governing equations with these constraints can now be derived from applying contact forces (24-25) to (12), (13) and (15), along with using constraint equations (22-23).

$$-I_3\ddot{\theta}_3+m_3l_3\frac{\sin(\theta_3-\ \alpha_1)}{2\cos\alpha_1}\ddot{X}_{G_3}+\left[m_4l_3\frac{\sin(\theta_3-\ \alpha_1)}{2\cos\alpha_1}+m_4l_3\frac{\sin(\theta_3-\ \alpha_2)}{2\cos\alpha_2}\right]\ddot{x}=0 \tag{26}$$

$$m_3\tan\alpha_1\ddot{X}_{G_3}-m_3\ddot{Y}_{G_3}+m_4(\tan\alpha_1-\tan\alpha_2)\ddot{x}-m_3g=0 \tag{27}$$

$$(m_2a_2^2l_2^2\ +I_2)\ddot{\theta}_2-m_3l_2\frac{\sin(\theta_2-\ \alpha_1)}{\cos\alpha_1}\ddot{X}_{G_3}-m_4l_2\frac{\sin(\theta_2-\ \alpha_1)}{\cos\alpha_1}\ \ddot{x}+m_2gl_2a_2\cos\theta_2=M \tag{28}$$

$$\begin{aligned}
&\left[2X_{G_3}l_2\sin\theta_2-2Y_{G_3}l_2\cos\theta_2-2l_2l_3a_3\sin(\theta_2-\theta_3)\right]\ddot{\theta}_2\\
&+\left[2X_{G_3}l_3a_3\sin\theta_3-2Y_{G_3}l_3a_3\cos\theta_3+2l_2l_3a_3\sin(\theta_2-\theta_3)\right]\ddot{\theta}_3\\
&+\left[2X_{G_3}-2\ (l_2\cos\theta_2+l_3a_3\cos\theta_3)\right]\ddot{X}_{G_3}\\
&+\left[2Y_{G_3}-2(l_2\sin\theta_2+\ l_3a_3\sin\theta_3)\right]\ddot{Y}_{G_3}+\ 2\dot{X}_{G_3}^2+\ 2\dot{Y}_{G_3}^2\\
&+4\dot{X}_{G_3}\left(l_2\dot{\theta}_2\sin\theta_2+\ l_3a_3\ \dot{\theta}_3\sin\theta_3\right)\\
&+2X_{G_3}\left(l_2\dot{\theta}_2^2\cos\theta_2+l_3a_3\dot{\theta}_3^2\cos\theta_3\right)\\
&-4\dot{Y}_{G_3}\left(l_2\dot{\theta}_2\cos\theta_2+\ l_3a_3\dot{\theta}_3\cos\theta_3\right)\\
&+2Y_{G_3}\left(l_2\dot{\theta}_2^2\sin\theta_2+l_3a_3\dot{\theta}_3^2\sin\theta_3\right)\\
&-2l_2l_3a_3\left(\dot{\theta}_2-\ \dot{\theta}_3\right)^2\ \cos(\theta_2-\ \theta_3)=0
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\left[\ 2x\ (1-a_3)l_3\,sin\,\theta_3-2X_{G_3}(1-a_3)l_3\,sin\,\theta_3+2Y_{G_3}(1-a_3)l_3\,cos\,\theta_3\right]\ddot{\theta}_3\\
&+\left[2X_{G_3}-\ 2x+2\ (1-a_3)l_3\,cos\,\theta_3\right]\ddot{X}_{G_3}\\
&+\left[2Y_{G_3}+2(1-a_3)l_3\,sin\,\theta_3\right]\ddot{Y}_{G_3}\\
&+\left[2x-2X_{G_3}-2\ (1-a_3)l_3\,cos\,\theta_3\ \right]\ddot{x}\ +2\dot{x}^2+\ 2\dot{X}_{G_3}^2+\ 2\dot{Y}_{G_3}^2-\ 4\dot{x}\dot{X}_{G_3}\\
&+4\dot{x}(1-a_3)l_3\dot{\theta}_3\,sin\,\theta_3\ +2x\ (1-a_3)l_3\dot{\theta}_3^2\,cos\,\theta_3\\
&-4\dot{X}_{G_3}(1-a_3)l_3\dot{\theta}_3\,sin\,\theta_3-\ 2X_{G_3}l_3\dot{\theta}_3^2\cos\theta_3\ (1-a_3)\\
&+4\dot{Y}_{G_3}l_3\dot{\theta}_3\cos\theta_3\ (1-a_3)-\ 2Y_{G_3}l_3\dot{\theta}_3^2\sin\theta_3\ (1-a_3)=\ 0
\end{aligned} \tag{30}$$

3. Simulations

As it was mentioned in the in section 1, non-conservative contact forces Q_{c_i} are exerted during continuous contact mode to crank, connecting rod and the slider. Q_c is highly dependent on the magnitude of initial impact velocity $\dot{\delta}_\mathrm{i}(-)$, a parameter that should be updated at start of each mode of continuous contact. In current study, it is proposed that the term $\Delta = \prod_{i=1}^{N}(r_i - (R_{B_i} - R_{J_i})$ should be studied for detecting the beginning of each continuous contact mode, in which "N" is the number of clearance joints. Each change in the sign of Δ denotes continuous contact mode has started or ended in one of clearance joints. If a change occurs, clearance radius in each joint immediately after transition is calculated ($r_{i_{end}}$ at t_{end}) and compared with values before impact. If $r_{i_{end-1}} \le (R_{B_i} - R_{J_i})$ and $r_{i_{end}} \ge (R_{B_i} - R_{J_i})$; at some point in between t_{end-1} and t_{end} in joint i continuous contact mode has started and the term $\dot{\delta}_\mathrm{i}(-)$ should be updated. Initial conditions are assumed to be that of an ideal mechanism without any clearance. Crank angular velocity is set to 5000 rpm. In case of multiple clearances, 'first clearance joint' refers to the joint between crank and connecting rod and 'second clearance joint' refers to joint between connecting rod and slider. If it is not mentioned otherwise, physical and geometric properties of components and clearance properties listed in tables 1 and 2 (case A) are used.

Table 1: Physical and geometric properties of the bodi

Body no.	Length (m)	Width (m)	Height (m)	Width (m)	a
2	0.05	0.015	0.01875	0.3	0.5
3	0.12	0.015	0.01875	021	0.5
4	–	0.14	–	0.14	–

Table 2: Clearance joints' properties

Clearance joint no.	R_B	Clearance size	C_e	Young's Modulus	Poisson's ratio	Friction coefficient		
						Case A	Case B	Case C
1	10 mm	0.5 mm	0.9	207 GPa	0.3	00	00.2	0.4
2	10 mm	0.5 mm	0.9	207 GPa	0.3	00	00.2	0.4
Single clearance	10 mm	0.5 mm	0.9	207 GPa	0.3	00	00.2	0.4

Figure (4) depicts the trajectory of the journal centre relative to the bearing in various clearance joints. Since the mechanism is set to start its motion from the initial conditions akin to the motion of an ideal mechanism, the journal centre trajectories relative to the bearings start from the centre of the clearance circle.

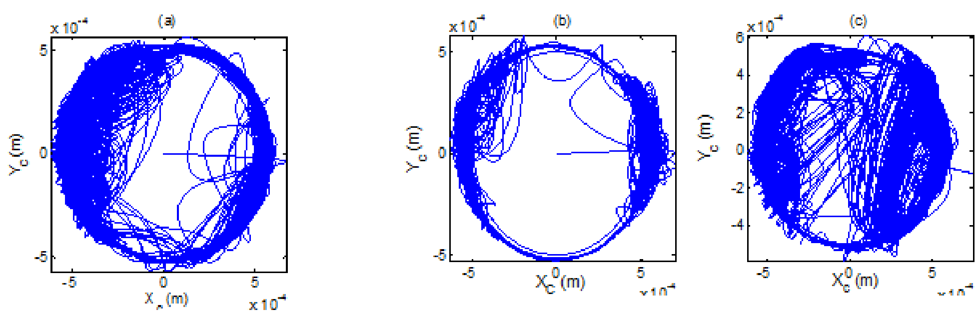

Fig 4: Journal centre trajectory relative to bearing in mechanism (a) single and clearance and in mechanism with multiple clearances in (b) first clearance joint (c) second clearance joint.

The path of journal centre in the clearance joint between connecting rod and slider is highly unpredictable. Free flight mode can be observed frequently, as a result contact force is highly discontinuous and system's behaviour is further from desired state. Perturbations in responses of the mechanisms with clearance joints can be observed from Figure (5), which depicts velocity and acceleration of the slider for mechanisms with ideal joints, single clearance joint and multiple clearance joints. Not only the mechanism with multiple clearance joints exhibit higher peaks than mechanism with a single clearance joint and ideal mechanism; phase difference between its response and dynamical behaviour of ideal mechanism is of greater value as well, compared to phase difference of the mechanisms with single clearance joint and with ideal joints.

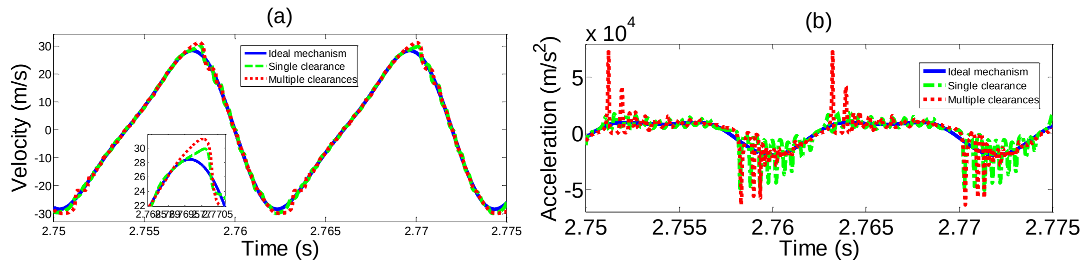

Fig 5: Slider's (a)velocity (b) acceleration for mechanisms with clearance joints

For the mentioned parameters, the mechanism will settle into periodic motion. However this would not be the case for all configurations and mechanisms with clearances frequently show chaotic behaviours in various situations. This is depicted in the bifurcation diagrams of Figure (6), which was drawn, based on Poincare maps for varying clearance sizes in mechanism with multiple clearances, using points in which slider's velocity is equal to zero and its sign changes. The system undergoes complex, chaotic behaviour for values of clearance around 0.5 mm (Figures (6-a) and (6-b)). Moreover, as it can be observed from Figures (6-c) and (6-d), system generally becomes more chaotic with increasing clearance size, and Poincare maps of Figure (7) become less concentrated

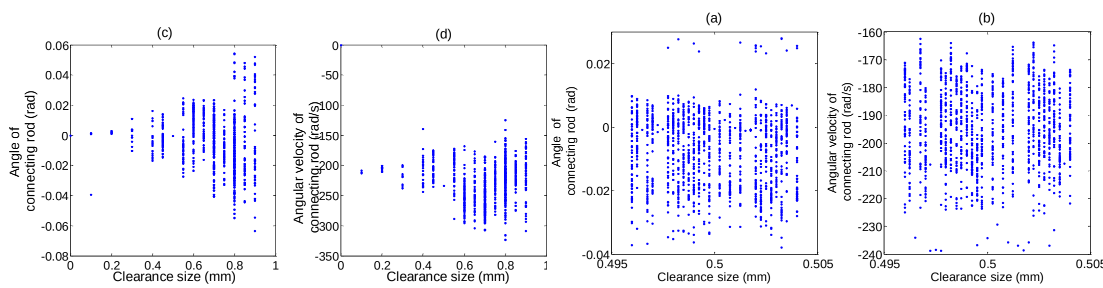

Fig 6: Bifurcation diagram, (a) θ_3 (b) $\dot{\theta}_3$ for clearance sizes between 0.496-0.504 mm and (c) θ_3 (d) $\dot{\theta}_3$ for varying clearance sizes between 0 – 1 mm.

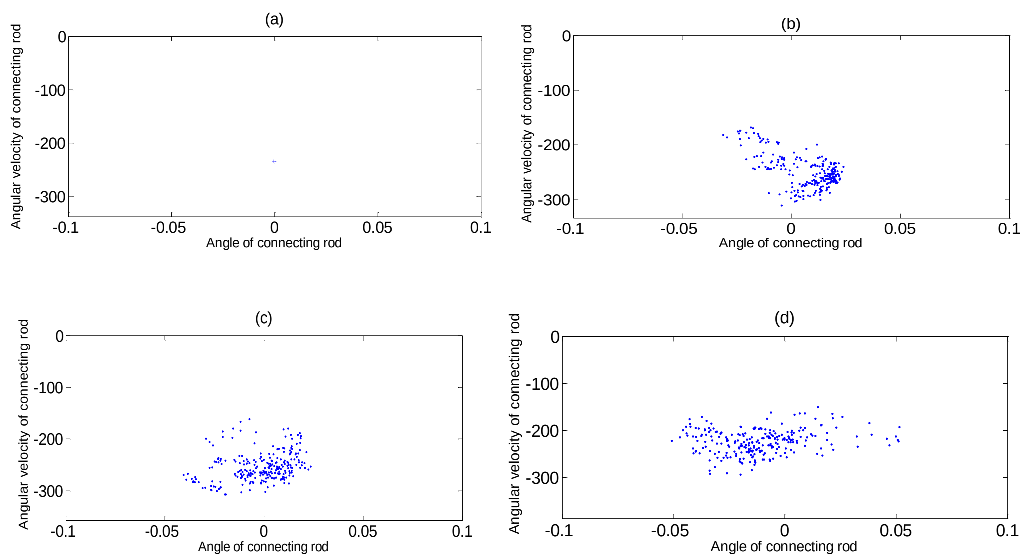

Fig 7: Poincare map for clearance size of (a) 0.5 mm (b) 0.6 mm (c) 0.7 mm (d) 0.8 mm

Usually friction is ignored in numerical simulations of mechanisms with joint clearances because of the complexities it proposes, but considering its effect will result in notably more accurate results. To include the effects of friction, equations (10), (12-13) and (14-15) of section 2.1 should be used as well. It is important to choose appropriate values for v_0 and υ_1, such that maximum possible accuracy of results can be obtained without numerically diverging the calculation or reducing time step. Since friction causes an increase in the general stiffness of the system, higher contact forces and accelerations are expected (Figure (8)), the value of which further increase with increasing friction coefficients. Moreover, with increasing joint friction force, the system's path becomes more and more chaotic, which can be observed from phase diagrams of Figure (9).

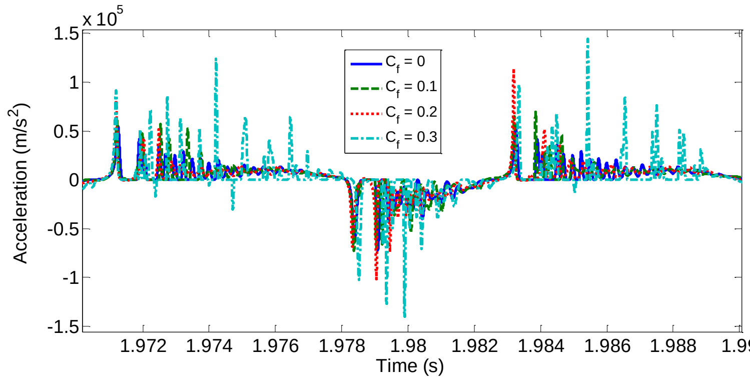

Fig 8: Slider acceleration for various friction coefficients

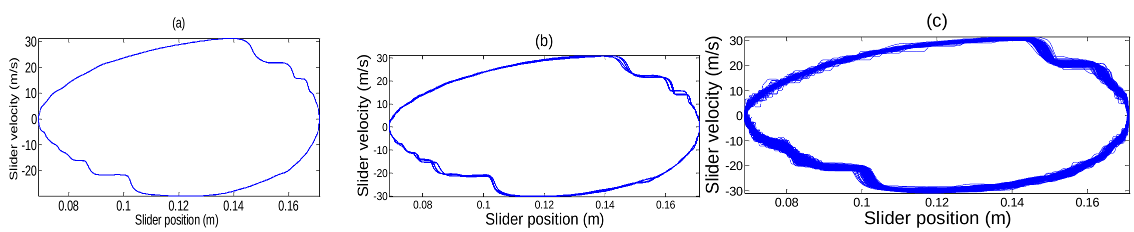

Fig 9: Phase diagram for steady state of mechanism with clearance size 0.5 mm and (a) no friction in joints (b) friction coefficient of 0.2 (c) friction coefficient of 0.4.

From joint centre trajectories of Figure 4, it's obvious that the amount of impacts taking place in a clearance joint during the motion of mechanism is extraordinarily high. Extra noise, performance deterioration over time because of fatigue and lower positional accuracy can be pointed out as consequences of this behaviour. Preventing contact loss in joints of the crank-slider mechanism was proposed in this study in section 2.3 as a method to overcome or reduce aforementioned problems. By applying desired properties of motion, the mechanism is simulated and required input torques for desired motions is obtained. The results for two cases are presented in Figures (10) and (11).

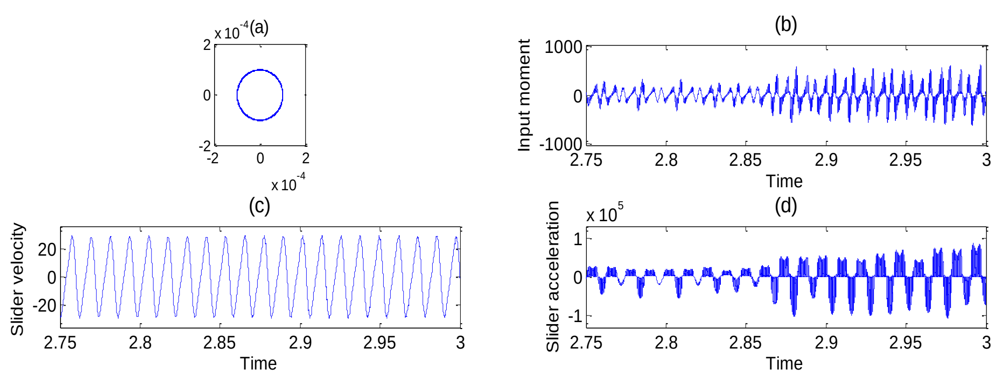

Fig 10: Mechanism with single clearance joint (a) joint center trajectory relative to bearing (b) input torque (c) slider velocity (d) slider acceleration.

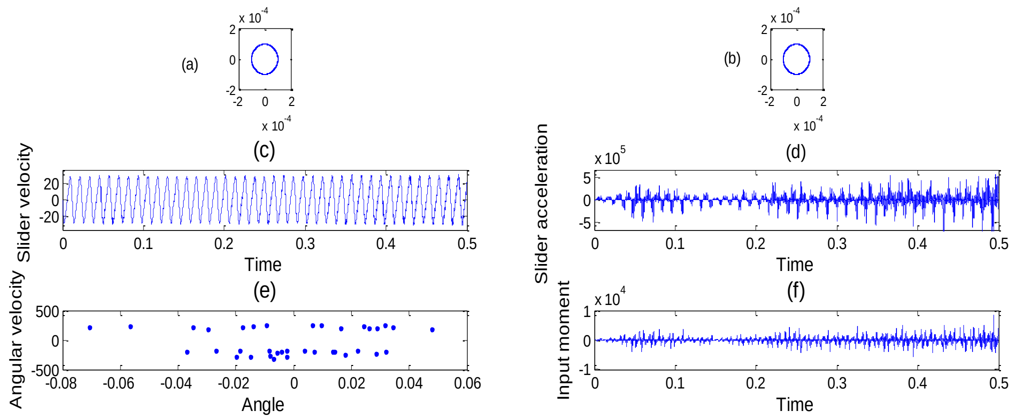

Fig 11: Mechanism with multiple clearance joints (a) joint center trajectory for first joint (b) joint center trajectory for second joint (c) slider velocity (d) slider acceleration (e) Poincare map (f) input torque .

It is observed that continuous contact has been established in both cases. Control scheme has been successfully applied and therefore desired effects are achieved. It is observed that the number of points on Poincare maps is higher and therefore the frequency of oscillations decrease. Since no impact takes place in the clearance joints, fatigue is reduced and reduction in mechanism's noise is expected. Results also indicate that extra input torque is needed for controlling mechanism. Under successful control, the mechanism exhibits improved behaviour and some of undesired effects resulting from clearance joints in the crank-slider mechanism have been prevented or reduced.

4. Conclusions

Based on results of numerical simulations, it was observed that joint clearance causes perturbations in response of the crank-slider mechanism. From comparisons between responses of crank-slider mechanism with a single clearance joint and multiple clearance joints, it is concluded that perturbations intensify as number of clearance joints in mechanism increases. Nonlinear dynamics of system were analysed using Poincare maps and bifurcation diagrams. The system exhibits chaotic motion under a host of various configurations. Effects of joint friction on dynamics of system with joint clearances have been studied, and it has been observed that with the introduction of joint friction, the mechanism no longer exhibits periodic motion and nonlinear dynamics and multiple phase paths are observed. To prevent discontinuous forces in joints and avoiding its undesired effects, a control scheme was proposed to guide the mechanism under the constraint of continuous contact. Control method has been applied and numerically simulated, and it was observed that unwanted effects of clearance in mechanism have been reduced or overcame

References

1. S. W. Earles, C .L. S. Wu, Motion analysis of a rigid link mechanism with clearance at a bearing using Lagrangian mechanics and digital computation, Mechanisms, (1973) 83–89.

2. Erkayaa, and İ. Uzmay, Experimental Investigation of Joint Clearance Effects, *Multibody System Dynamics*, 24 (2010) 81-102.

3. M. Dupac, and D. G. Beale, Dynamic Analysis of a Flexible Linkage Mechanism with Cracks and Clearance, *Mechanism and Machine Theory*, 45 (2010) 1909–1923.

4. S. Dubowsky, On predicting the dynamic effects of clearances in planar mechanisms, Journal of Engineering for Industry, (1974) 317–323.

5. H. Hertz, Über die Berührung fester elastischer Körper, Journal reine und angewandte, Mathematik, (1881) 92.

6. H. M Lankarani, P. E. Nikravesh, A contact force model with hysteresis damping for im-pact analysis of multibody systems, Journal of Mechanical Design 376 (1990)112, 369.

7. C.S. Koshy, P. Flores, and H.M. Lankarani, Study of the effect of contact force model on the dynamic response of mechanical systems with dry clearance joints: computational and experimental approaches, *Nonlinear Dynamics*, 73 (2013) 325–338.

8. Houfek, L. Houfek, Bifurcation And Chaos In Couplings With Clearance, *Advanced Materials Research*, 936 (2014) 2120-2124.

9. J. A. C. Ambrósio, Impact of Rigid and Flexible Multibody Systems: Deformation De-scription and Contact Models, Virtual Nonlinear Multibody Systems, (2002) 15-33.

10. A. A. Olyaei, M. R. Ghazavi, Stabilizing slider-crank mechanism with clearance joints, Mechanism and Machine Theory 53, (2012)17-29

11. Y.S. Yoon, J.S. Shim, S.B. Lee, K. D. Willmert, Reduction of the joint clearance effect for a planar flexible mechanism, Journal of Mechanical Science and Technology 9, (1995) 312–322.

12. P. Flores, and H. M. Lankarani, Dynamic Response of Multibody Systems with Multiple Clearance Joints, *Journal of Computational and Nonlinear Dynamics*, 7(3) (2012) 031003-13.

13. L. Cheng, T. Qiang, and H. Haiyan, Dynamics and Control of a Spatial Rigid-Flexible Multibody System with Multiple Cylindrical Clearance Joints, *Mechanism and Machine Theory*, 52 (2012) 106-129.