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A theoretical approach for flexural behavior of FG vibrating micro-plates with piezoelectric layers considering a hybrid length scale parameter

Korosh Khorshidi[a*], Mahdi Bahrami[b], Mahdi Karimi[b], Mohsen Ghasemi[b]

[a]*Associated Professor, Department of Mechanical Engineering, Faculty of Engineering, Arak University, Arak, Iran*
[b] *MSc Student, Department of Mechanical Engineering, Faculty of Engineering, Arak University, Arak, Iran*

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ABSTRACT

In the current study, the mechanical performance of functionally graded oscillating micro-plates bonded with piezoelectric layers is examined using the modified couple stress theory. The modified couple stress theory contains a length scale parameter that considers the size-effects of micro-plates. The various modified shear deformation theories are employed to represent the displacement field of micro-plate, such as exponential, parabolic, hyperbolic, trigonometric, and fifth-order shear deformation theories. The properties of FG micro-plate, such as Young's modulus, density, and length scale parameter, are assumed to vary smoothly and across the micro-plate thickness based on the Power-law model. The governing equations of motion are obtained by Hamilton's principle and solved by a theoretical approach under various boundary conditions. The accuracy of the proposed model is validated based on a comparison of the results with the accepted studies. Computational analysis is carried out to clarify the impacts of mechanical and geometrical variables on the natural frequencies of micro-plates.

1. Introduction

Functionally graded materials (FGMs) have drawn the extensive attention of scholars in recent decades due to their favorable characteristics[1-3]. Because of the less weight to strength ratio and appropriate thermal and corrosion resistance, these materials have been used in many engineering applications such as turbine blades, rocket nozzle, combustion engine, etc. FGMs are made of two or more substances whose characteristics vary continuously from one direction to

* Corresponding author:
E-mail address: k-khorshidi@araku.ac.ir (K. Khorshidi)

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another face. In the mid-1980s the fundamental researches on the FGMs accomplished by a group of scientists[4]. Since then, numerous studies and investigations are devoted to functionally graded materials[5-7]. Modified couple stress theory [8] and nonlocal elasticity theory[9] are assumed to analyze the behavior of functionally graded rectangular nanostructures. Fallah and Khorshidi [10] analyzed the effect of nonlinear temperature distribution on the free vibration analysis of functionally graded laminated plates considering shear deformation effects. The results illustrated that the inclining of the FG index causes a reduction in the frequencies. Khorshidi and Fallah[11] investigated the buckling response of functionally graded nano-plates using nonlocal theory and exponential shear deformation formulation.

Nowadays, the mechanical behavior of the nano and micro-structures due to their excellent properties are considered by many scientists[12-15]. Since the classical continuum approach is unable to analyze the mechanical behavior of the nano and micro-structures perfectly, various non-classical models have been proposed[16-19]. Yang *et al*. [20] demonstrated that the strain energy is a function of the curvature gradient tensor as well as the strain tensor. They presented the modified couple stress theory possessing only a material length scale parameter. Li and Pan [21] presented an analytical model to study the dynamic and bending performance of FG piezoelectric micro-plates capturing the inertia effects. The results illustrated the resonance frequencies decline with increasing the FG parameter. Khorshidi and Karimi[22] introduced an analytical response to calculate the resonance frequencies of the piezoelectric nano-plates using nonlocal elasticity theory in contact with the sloshing liquid. A varied number of plate theories have been used for capturing the equations of motion. Sahmani and Aghdam [23] applied the nonlocal elasticity theory to examine the instability of FG nano-panels with piezoelectric facesheets. Abazid and Sobhy[24] studied the bending of FG piezoelectric micro-plates resting on Pasternak foundation considering the couple stress resultants. The equations of the model were obtained considering an external load, electric voltage, and thermal environment.

Piezoelectric materials are a class of smart materials with direct effect and converse effect. The direct effect is referred to as generate electrical voltage due to applied pressure or stress. Also, the produced deformation due to the applied electric field is called the converse effect. Recently, functionally graded structures integrated with piezoelectric layers are widely used for stability and self-controlling systems such as microgrippers [25-29]. Reddy and Cheng[30] studied the static analysis of FG plates surrounded by a piezoelectric actuator undergoing the thermomechanical loads. They observed that the volume fraction distribution is significant concerning the applied thermal loading. Li *et al*.[31] utilized the piezoelectric material to control the oscillation of FG plates using the classical laminated plate hypothesis. They concluded that using the piezoelectric material in the FG structures can decrease the excessive vibration of the system. Selim *et al*.[32] applied Reddy’s higher-order theory to analyze the control of FG vibrating plates enhanced with the piezoelectric layers. It was found that the placement of piezoelectric layers on two opposite sides of FGM plates is useful for better vibration control. Arefi *et al*. [33] employed a theoretical method for oscillation analysis of a three-layer nano-plate with FG core and piezo-magnetics layers resting on the Pasternak foundation. The proposed model was grounded on the nonlocal elasticity theory and Mindlin theory. Ebrahimi and Rastgo investigated the free vibration behavior of FG circular [34] and annular [35] plates with two piezoelectric patches based on the classical plate theory. Karami *et al*. [36] considered a second-order theory to study the thermal buckling of a functionally graded nano-plate bonded with piezoelectric patches embedded on Pasternak-Winkler foundation on the basis of the nonlocal elasticity theory. They observed that utilizing piezoelectric in functionally graded nano-plate lowers the critical buckling temperature of the FG nano-structure.

From the literature review, it can be seen that assuming constant length scale parameters results in imprecise results. To be more accurate, it is recommended to use a variable small scale parameter. This study aims to propose a theoretical response for FG vibrating micro-plates with two piezoelectric layers capturing the couple stress resultants. Various modified plate theories are contemplated to express the kinematic of the micro-plate. The partial differential equations associated with the micro-plate are obtained by Hamilton's principle and then discretized by the Galerkin approach. Also, the impacts of length scale parameter, geometrical ratios, volume fraction index, and various boundary conditions are investigated.

2. Formulation

A functionally graded micro-plate which is a in length, b in width and $2h$ in thickness with two piezoelectric layers is considered as shown in Figure 1. It is presumed that the properties of functionally graded micro-plate alter smoothly through the thickness direction. Also, the effective properties of functionally graded micro-plate such as Young's modulus, length scale parameter, and density are estimated according to the Power-law model as follows:

$$\begin{aligned} E_{(z)} &= E_m + (E_c - E_m)\, V_m \\ \ell_{(z)} &= \ell_m + (\ell_c - \ell_m)\, V_m \\ \rho_{(z)} &= \rho_m + (\rho_c - \rho_m)\, V_m \end{aligned} \tag{1}$$

in which the subscripts m and c express the metallic and ceramic constituents, respectively. Also, the metal volume fraction V_m can be expressed as:

$$\begin{aligned} V_m &= \left(\frac{1}{2} + \frac{z}{h}\right)^{\alpha} \\ V_m + V_c &= 1 \end{aligned} \tag{2}$$

where V_c and α are the volume fraction of ceramic and the volume fraction index, respectively.

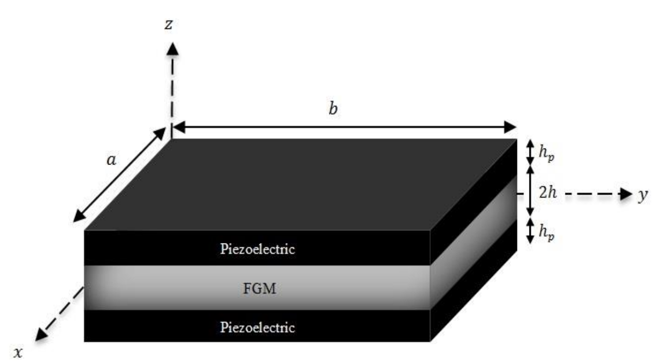


Fig 1. Schematic of FG micro-plate bonded with piezoelectric layers.

3. Modified couple stress theory

Yang *et al.*[20] illustrated that the symmetric part of the curvature gradient tensor and the strain tensor play an influential role in the value of the strain energy density. They proposed the modified couple stress theory in which contains a single length scale variable ℓ. According to this theory the strain energy U, in an isotropic structure with volume V is given by:

$$U = \frac{1}{2}\int_V (\sigma_{ij}\,\varepsilon_{ij} + m_{ij}\,\chi_{ij})\ dV \tag{3}$$

where σ is the stress tensor, ε is the strain tensor, m is the deviatoric part of the couple stress tensor, and χ is the symmetric curvature tensor. Also, the strain tensor and symmetric curvature tensor are given as:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4-a}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{4-b}$$

where u_i and θ_i are displacement vectors and rotation vectors, respectively. The rotation vectors θ_i are defined as:

$$\theta_i = \frac{1}{2}u_{k,j}e_{ijk} \tag{5}$$

where e_{ijk} is the permutation symbol. The constitutive relations of FG micro-plate bonded with two piezoelectric layers are written as:

$$\begin{aligned}\{\sigma\} &= [Q]\{\varepsilon\} - [e]\{E\} \\ \{D\} &= [e]\{\varepsilon\} + [d]\{E\}\end{aligned} \tag{6}$$

where σ, D, Q, e, d, and ε are stress tensor, electric displacement vector, elastic coefficients matrix, piezoelectric coefficients matrix, dielectric coefficients matrix, and strain tensor, respectively. The elastic constants Q_{ij} can be expressed as:

$$\begin{aligned} Q_{11} = Q_{22} &= \frac{E_{(z)}}{1-\nu^2} \\ Q12 &= \nu\ Q11 \\ Q_{44} = Q_{55} = Q_{66} &= \frac{E_{(z)}}{2(1+\nu)} \end{aligned} \tag{7}$$

Also, the electric field vector E is obtained as follows:

$$E_i = -\,\Phi_{,i} \quad , i = x, y, z \tag{8}$$

where Φ is the electric potential field and the proposed relation[37] is given by:

$$\Phi_{(x,y,z,t)} = \begin{cases} \phi_{(x,y,t)}\left[1-(\frac{z-h-h_p/2}{h_p/2})^2\right] & h \le z \le h+h_p \\ \phi_{(x,y,t)}\left[1-(\frac{-z-h-h_p/2}{h_p/2})^2\right] & -h-h_p \le z \le -h \end{cases} \tag{9}$$

where $\phi_{(x,y,t)}$ is the electric potential distribution in the middle surface of the micro-plate.

4. Modified shear deformation theory

Within the framework of the modified shear deformation theory [37], the displacement field can be written as follows:

$$\begin{aligned} u\,(x,y,z,t) &= u_0\,(x,y,t) - z\frac{\partial w_0}{\partial x} + f_{(z)}\,\zeta(x,y,t) \\ v\,(x,y,z,t) &= v_0\,(x,y,t) - z\frac{\partial w_0}{\partial y} + f_{(z)}\,\psi(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{10}$$

where u, v, and w denote displacements along x, y, and z axis. ζ and ψ are the rotation functions of the middle plane in the x and y directions, respectively. Five different distributions for $f_{(z)}$ such as exponential theory (ESDT), trigonometric theory (TSDT), hyperbolic theory (HSDT), parabolic theory (PSDT), and fifth-order theory (FOSDT) are listed in Table 1.

Table 1. Different distributions for $\boldsymbol{f}_{(\boldsymbol{z})}$

Type of theory	$\boldsymbol{f}_{(\boldsymbol{z})}$
ESDT	$z\,e^{-2(\frac{z}{h})^2}$
TSDT	$\frac{h}{\pi}Sin(\frac{\pi z}{h})$
HSDT	$h\,Sinh\left(\frac{z}{h}\right) - z\,Cosh(\frac{1}{2})$
PSDT	$z(\frac{5}{4} - \frac{5z^2}{3h^2})$
FOSDT	$z(\frac{1}{h} - \frac{2z^2}{h^3} + \frac{8z^4}{5h^5})$

By considering the Eq. (4-a), the strain field can be obtained as follows:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f_{(z)}\frac{\partial \zeta}{\partial x}$$
$$\varepsilon_{yy} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2} + f_{(z)}\frac{\partial \psi}{\partial y}$$
$$\varepsilon_{xy} = \frac{1}{2}(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}) - z\frac{\partial^2 w_0}{\partial x \partial y} + \frac{1}{2}f_{(z)}(\frac{\partial \zeta}{\partial y} + \frac{\partial \psi}{\partial x}) \quad (11)$$
$$\varepsilon_{xz} = \frac{1}{2}(\frac{df_{(z)}}{dz}\zeta)$$
$$\varepsilon_{yz} = \frac{1}{2}(\frac{df_{(z)}}{dz}\psi)$$

According to Eq. (5), the rotation vector can be written as:

$$\theta_x = \frac{\partial w_0}{\partial y} - \frac{1}{2}(\frac{df_{(z)}}{dz}\psi)$$
$$\theta_y = \frac{1}{2}(\frac{df_{(z)}}{dz}\zeta) - \frac{\partial w_0}{\partial x} \quad (12)$$
$$\theta_z = \frac{1}{2}(\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}) + \frac{1}{2}f_{(Z)}(\frac{\partial \psi}{\partial x} - \frac{\partial \zeta}{\partial y})$$

The curvature components can be obtained from Eq. (4-b) as follows:

$$\chi_{xx} = \frac{\partial^2 w_0}{\partial x \partial y} - \frac{1}{2}(\frac{df_{(z)}}{dz}\frac{\partial \psi}{\partial x})$$
$$\chi_{yy} = \frac{1}{2}(\frac{df_{(z)}}{dz}\frac{\partial \zeta}{\partial y}) - \frac{\partial^2 w_0}{\partial x \partial y}$$
$$\chi_{zz} = \frac{1}{2}\frac{df_{(z)}}{dz}(\frac{\partial \psi}{\partial x} - \frac{\partial \zeta}{\partial y})$$
$$\chi_{xy} = \frac{1}{2}(\frac{\partial^2 w_0}{\partial y^2} - \frac{\partial^2 w_0}{\partial x^2}) + \frac{1}{4}\frac{df_{(z)}}{dz}(\frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial y}) \quad (13)$$
$$\chi_{xz} = \frac{1}{4}(\frac{\partial^2 v_0}{\partial x^2} + f_{(z)}\frac{\partial^2 \psi}{\partial x^2} - \frac{d^2 f_{(z)}}{dz^2}\psi)$$
$$\chi_{yz} = \frac{1}{4}(\frac{d^2 f_{(z)}}{dz^2}\zeta - \frac{\partial^2 u_0}{\partial y^2} - f_{(z)}\frac{\partial^2 \zeta}{\partial y^2})$$

5. Equations of motion

The governing equations can be obtained on the grounds of Hamilton’s principle as follows:

$$\int_0^t (\delta T + \delta W_e - \delta U)\ dt = 0 \quad (14)$$

where δ indicates the variation operator, T is the total kinetic energy, W_e is the energy of the external loadings and U represents the strain energy of micro-plate. The total kinetic energy T can be written as follows:

$$= \frac{1}{2} \int_V \rho_{(z)} \left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right) \, dV \tag{15}$$

By substituting Equations (3) and (15) into Eq. (14) and some mathematical simplifications, system of equations will be obtained in the following:

$$\begin{aligned}
&\delta u_0 : \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + \frac{1}{2}\left(\frac{\partial^2 R_{xz}}{\partial x \partial y} + \frac{\partial^2 R_{yz}}{\partial y^2}\right) = I_1 \frac{\partial^2 u_0}{\partial t^2} + I_4 \frac{\partial^3 w_0}{\partial x \partial t^2} + I_5 \frac{\partial^2 \zeta}{\partial t^2} \\
&\delta v_0 : \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} - \frac{1}{2}\left(\frac{\partial^2 R_{xz}}{\partial x^2} + \frac{\partial^2 R_{yz}}{\partial x \partial y}\right) = I_1 \frac{\partial^2 v_0}{\partial t^2} + I_4 \frac{\partial^3 w_0}{\partial y \partial t^2} + I_5 \frac{\partial^2 \psi}{\partial t^2} \\
&\delta w_0 : \frac{\partial^2 S_{xx}}{\partial x^2} + \frac{\partial^2 S_{yy}}{\partial y^2} + 2\frac{\partial^2 S_{xy}}{\partial x \partial y} - \frac{\partial T_{xz}}{\partial x} - \frac{\partial W_{xz}}{\partial x} - \frac{\partial T_{yz}}{\partial y} - \frac{\partial W_{yz}}{\partial y} \\
&\quad + \frac{1}{2}\left(\frac{\partial^2 R_{xx}}{\partial x \partial y} + \frac{\partial^2 r_{xx}}{\partial x \partial y} + \frac{\partial^2 r_{yy}}{\partial x \partial y} - \frac{\partial^2 R_{yy}}{\partial x \partial y} + \frac{\partial^2 R_{xy}}{\partial y^2} - \frac{\partial^2 r_{xy}}{\partial y^2} + \frac{\partial^2 r_{xy}}{\partial x^2} - \frac{\partial^2 R_{xy}}{\partial x^2}\right) \\
&\quad = I_1 \frac{\partial^2 w_0}{\partial t^2} - I_2 \frac{\partial^4 w_0}{\partial x^2 \partial t^2} - I_2 \frac{\partial^4 w_0}{\partial y^2 \partial t^2} - I_4 \frac{\partial^3 u_0}{\partial x \partial t^2} - I_4 \frac{\partial^3 v_0}{\partial y \partial t^2} - I_5 \left(\frac{\partial^3 \zeta}{\partial x \partial t^2} + \frac{\partial^3 \psi}{\partial y \partial t^2}\right) \\
&\delta \zeta : \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} - Q_{xz} + \frac{1}{2}\left(\frac{\partial F_{yy}}{\partial y} - \frac{\partial F_{zz}}{\partial y} + \frac{\partial F_{xy}}{\partial x} - G_{yz} + \frac{\partial^2 H_{xz}}{\partial x \partial y} + \frac{\partial^2 H_{yz}}{\partial y^2}\right) \\
&\quad = I_5 \frac{\partial^2 u_0}{\partial t^2} + I_6 \frac{\partial^3 w_0}{\partial x \partial t^2} + I_3 \frac{\partial^2 \zeta}{\partial t^2} \\
&\psi : \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{xy}}{\partial x} - Q_{yz} - \frac{1}{2}\left(\frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{zz}}{\partial x} + \frac{\partial F_{xy}}{\partial y} - G_{xz} + \frac{\partial^2 H_{xz}}{\partial x^2} + \frac{\partial^2 H_{yz}}{\partial x \partial y}\right) \\
&\quad = I_5 \frac{\partial^2 v_0}{\partial t^2} + I_6 \frac{\partial^3 w_0}{\partial y \partial t^2} + I_3 \frac{\partial^2 \psi}{\partial t^2} \\
&\delta \phi : \int_{-h-h_p}^{h+h_p} \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) dz = 0
\end{aligned} \tag{16}$$

where the stress resultants are defined as follows:

$$\begin{aligned}
\left(N_{xx}, N_{yy}, N_{xy}\right) &= \int_{-h-h_p}^{h+h_p} \left(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\right) dz \\
\left(S_{xx}, S_{yy}, S_{xy}\right) &= \int_{-h-h_p}^{h+h_p} \left(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\right) \times (-z) \, dz \\
\left(P_{xx}, P_{yy}, P_{xy}\right) &= \int_{-h-h_p}^{h+h_p} \left(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\right) \times f_{(z)} \, dz
\end{aligned} \tag{17}$$

$$(W_{xz}, W_{yz}) = \int_{-h-h_p}^{h+h_p} (\tau_{xz}, \tau_{yz})\, dz$$

$$(T_{xz}, T_{yz}) = \int_{-h-h_p}^{h+h_p} (-\tau_{xz}, -\tau_{yz})\, dz$$

$$(Q_{xz}, Q_{yz}) = \int_{-h-h_p}^{h+h_p} (\tau_{xz}, \tau_{yz}) \times \frac{df_{(z)}}{dz}\, dz$$

$$(R_{xx}, R_{yy}, R_{xy}) = \int_{-h-h_p}^{h+h_p} (m_{xx}, m_{yy}, m_{xy})\, dz$$

$$(R_{xz}, R_{yz}) = \int_{-h-h_p}^{h+h_p} (m_{xz}, m_{yz})\, dz$$

$$(r_{xx}, r_{yy}, r_{xy}) = \int_{-h-h_p}^{h+h_p} (-m_{xx}, -m_{yy}, -m_{xy})\, dz$$

$$(F_{xx}, F_{yy}, F_{xy}, F_{zz}) = \int_{-h-h_p}^{h+h_p} (m_{xx}, m_{yy}, m_{xy}, m_{zz}) \times \frac{df_{(z)}}{dz}\, dz$$

$$(G_{xz}, G_{yz}) = \int_{-h-h_p}^{h+h_p} (m_{xz}, m_{yz}) \times \frac{d^2 f_{(z)}}{dz^2}\, dz$$

$$(H_{xz}, H_{yz}) = \int_{-h-h_p}^{h+h_p} (m_{xz}, m_{yz}) \times f_{(z)}\, dz$$

The inertia terms are expressed as:

$$(I_1, I_2, I_3, I_4, I_5, I_6) = \int_{-h-h_p}^{h+h_p} \rho_{(z)} \times \left(1, z^2, f_{(z)}^2, -z, z, -zf_{(z)}\right) dz \qquad (18)$$

6. Analytical solution

To obtain the theoretical solution of the assumed FG vibrating micro-plate for various boundary conditions, the Galerkin approach is considered. According to this method, the displacement functions u_0, v_0, w_0, ζ and ψ are expressed as follows:

$$
\begin{aligned}
u_0(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} u_{m,n}\, \frac{\partial X_m(x)}{\partial x} Y_n(y)\, e^{i\omega t} \\
v_0(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} v_{m,n}\, X_m(x) \frac{\partial Y_n(y)}{\partial y}\, e^{i\omega t} \\
w_0(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} w_{m,n}\, X_m(x) Y_n(y) e^{i\omega t} \\
\zeta(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} \zeta_{m,n}\, \frac{\partial X_m(x)}{\partial x} Y_n(y)\, e^{i\omega t} \\
\psi(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{m,n}\, X_m(x) \frac{\partial Y_n(y)}{\partial y}\, e^{i\omega t} \\
\phi(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} \phi_{m,n}\, X_m(x) Y_n(y)\, e^{i\omega t}
\end{aligned}
\tag{19}
$$

where $i=\sqrt{-1}$, M and N denote the order of series and ω is the natural frequency. $u_{m,n}$, $v_{m,n}$, $w_{m,n}$, $\zeta_{m,n}$, $\psi_{m,n}$ and $\phi_{m,n}$ are unknown coefficients which will be determined. The various distributions of $X_m(x)$ and $Y_n(y)$ for SSSS, CSSS, CSCS, CCSS, and CCCC boundary conditions are shown in Table 2. In the present work, M=N=6 is considered for all calculations.

7. Numerical results

In this paragraph, the computational results are presented for FG micro-plate for various boundary conditions. The properties of FG micro-plate alter smoothly across the thickness from the bottom to the top surface. The bottom and top surfaces are considered as Aluminum and Alumina (Al_2O_3), respectively. The piezoelectric layers are made of PZT-4. The values of material properties are presented in Table 3. The thickness is assumed to be $h = 17.6\ \mu m$ in all mathematical calculations.

Firstly, to confirm the precision of the presented model, the obtained data are compared with those of Ke *et al*. [38] and Thai and Kim [39]. In Table 4, the first three dimensionless natural frequencies of a piezoelectric rectangular plate are reported for SSSS, CCSS and CCCC boundary conditions. It is assumed that the piezoelectric material is PZT-4 and $\Delta T = 0$. Furthermore, a good comparison is presented for various modified plate theories with the Mindlin plate theory in Table 4.

Table 2. Acceptable values of $\mathbf{X_m(x)}$ and $\mathbf{Y_n(y)}$

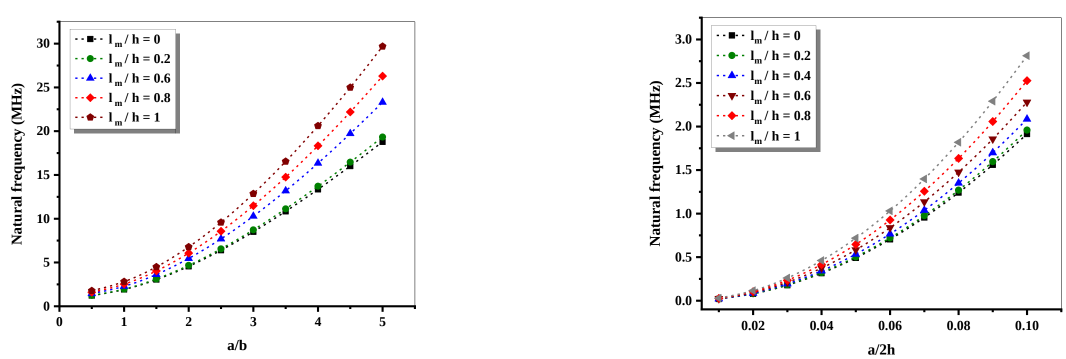
	Boundary Conditions		**Functions $X_{(x)}$ and $Y_{(y)}$**	
	$At \begin{cases} x=0 \\ x=a \end{cases}$	$At \begin{cases} y=0 \\ x=b \end{cases}$	$X_m(x)$	$Y_n(y)$
SSSS	$X_m(0)=X_m''(0)=0$ $X_m(a)=X_m''(a)=0$	$Y_n(0)=Y_n''(0)=0$ $Y_n(b)=Y_n''(b)=0$	$\sin(\frac{m\pi x}{a})$	$\sin(\frac{n\pi y}{b})$
CSSS	$X_m(0)=X_m'(0)=0$ $X_m(a)=X_m''(a)=0$	$Y_n(0)=Y_n''(0)=0$ $Y_n(b)=Y_n''(b)=0$	$\sin(\frac{m\pi x}{a})\left[\mathrm{Cos}\left(\frac{m\pi x}{a}\right)-1\right]$	$\sin(\frac{n\pi y}{b})$
CSCS	$X_m(0)=X_m'(0)=0$ $X_m(a)=X_m''(a)=0$	$Y_n(0)=Y_n'(0)=0$ $Y_n(b)=Y_n''(b)=0$	$\sin(\frac{m\pi x}{a})\left[\mathrm{Cos}\left(\frac{m\pi x}{a}\right)-1\right]$	$\sin(\frac{n\pi y}{b})\left[\mathrm{Cos}\left(\frac{n\pi y}{b}\right)-1\right]$
CCSS	$X_m(0)=X_m'(0)=0$ $X_m(a)=X_m'(a)=0$	$Y_n(0)=Y_n''(0)=0$ $Y_n(b)=Y_n''(b)=0$	$\sin^2(\frac{m\pi x}{a})$	$\sin(\frac{n\pi y}{b})$
CCCC	$X_m(0)=X_m''(0)=0$ $X_m(a)=X_m'(a)=0$	$Y_n(0)=Y_n''(0)=0$ $Y_n(b)=Y_n'(b)=0$	$\sin^2(\frac{m\pi x}{a})$	$\sin^2(\frac{n\pi y}{b})$

In Table 5, the dimensionless natural frequencies $\overline{\omega}=\omega\, a^2/h\sqrt{\rho_c/E_c}$ of functionally graded micro-plate are presented for various length scale ratios (ℓ/h), length to thickness ratios (a/2h), and volume fraction index (α) including 0, 1, 2, 5, and 10. Note that Thai and Kim [39] are reported their results for an FG micro-plate based on Reddy's plate theory. As shown in Table 5, length scale ratios have a noticeable impact on the resonance frequencies. It is observed that an increase in the value of the length scale ratio will raise the stiffness of functionally graded micro-plate, and consequently causes a notable increase in dimensionless frequencies. Also, it can be clearly observed that the highest values of dimensionless frequencies correspond to lower values of the FG parameter. In other words, the dimensionless frequencies experience a prominent reduction as the volume fraction index is inclined.

Table 3. Properties of constituent materials[40]

Property	FGM core plate		Piezoelectric layer
	Al	Alumina	PZT-4
E (GPa)	70	320.24	-
ν	0.3	0.2600	-
$\rho\ (Kg/m^3)$	2707	3750	7500
$C_{11}\ (GPa)$	-	-	132
$C_{12}\ (GPa)$	-	-	71
$C_{13}\ (GPa)$	-	-	73
$C_{33}\ (GPa)$	-	-	115
$C_{55}\ (GPa)$	-	-	26
$e_{31}\ (C/m^2)$	-	-	-4.1
$e_{15}\ (C/m^2)$	-	-	10.5
$e_{33}\ (C/m^2)$	-	-	14.1
$d_{11}\ (nF/m)$	-	-	7.124
$d_{33}\ (nF/m)$	-	-	5.841

Figure 2 shows the variations of the frequencies of the FG micro-plate equipped with piezoelectric layers as a function of the geometrical ratios for simply supported boundary conditions and $\alpha = 1$. In Figure 2a, it is assumed a/2h=0.1 and h_p/2h=0.05. In Figure 2b the aspect ratio a/b=1 and thickness ratio h_p/2h=0.05 are assumed. The variations are plotted versus different length scale parameters of metal to thickness ratios ℓ_m/h. A rise in the length to thickness ratio results in the increase of the micro-plate stiffness, and consequently the natural frequencies will be raised. It should be told that the length scale parameter $\ell_m/h = 0$ refers to the classical plate theory without the couple stress effect. As shown in Figure 2, the natural frequency rises by increasing the thickness and aspect ratio. For a micro-plate, with decreasing the width, the degrees of freedom and natural frequencies will increase.



(a) (b)

Fig 2. The natural frequency of the FG micro-plate with two piezoelectric layers for SSSS boundary condition versus a) various aspect ratios (a/b) and b) various thickness ratios (a/2h)

Table 4. Comparison of the first three dimensionless natural frequencies of the piezoelectric plate for SSSS, CCSS, and CCCC boundary conditions with ref. [38]

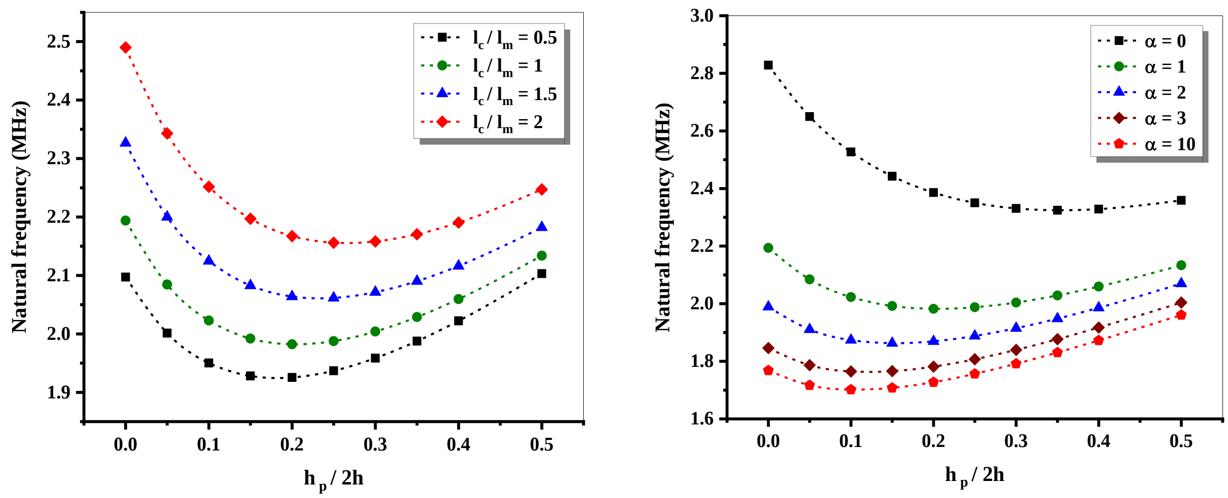
Boundary Condition	Method	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
SSSS	Present (ESDT)	0.5463	1.0143	1.5620
	Present (TSDT)	0.5462	1.0142	1.5619
	Present (HSDT)	0.5461	1.0141	1.5617
	Present (PSDT)	0.5461	1.0141	1.5617
	Present (FOSDT)	0.5641	1.0144	1.5619
	Ref [38]	0.5453	1.0132	1.5594
CCSS	Present (ESDT)	0.7192	1.1854	1.7881
	Present (TSDT)	0.7191	1.1853	1.7880
	Present (HSDT)	0.7190	1.1851	1.7880
	Present (PSDT)	0.7190	1.1851	1.7880
	Present (FOSDT)	0.7190	1.1854	1.7883
	Ref [38]	0.7184	1.1838	1.7868
CCCC	Present (ESDT)	0.9146	1.3691	2.0143
	Present (TSDT)	0.9142	1.3690	2.0141
	Present (HSDT)	0.9141	1.3689	2.0140
	Present (PSDT)	0.9141	1.3689	2.0140
	Present (FOSDT)	0.9142	1.3692	2.0143
	Ref [38]	0.9137	1.3672	2.0096

In Figure 3, the resonance frequency versus thickness ratio $h_p/2h$ is presented. The variations are plotted for assumptions of a/b=1 and a/2h=0.1. It is observed that frequency firstly decreases and then increases. The Young's modulus and density of the piezoelectric layers are lower and larger than the FG core plate, respectively. Hence, by raising the thickness of piezoelectric layers, the effective density increases and Young's modulus of the micro-plate decreases, resulting in the reduction of the vibrating frequencies. Also, the bending rigidity of the micro-plate depends on the thickness. Thus, as the thickness of piezoelectric layers increases, the stiffness of micro-plate and dimensionless frequencies will be raised. As shown in Figure 3a, by increasing the length scale parameter of ceramic, the frequency inclines. This is due to the fact that by increasing the length scale parameter of ceramic, the micro-plate will be stiffer. In Figure 3b, an increase in the FG index tends to induce a decrease in the frequencies. A higher volume fraction index means a higher volume fraction of ceramic. As the ceramic material has a higher density, so, the resonance frequency is lowered by increasing the volume fraction of ceramic.

Table 5. Comparison of the dimensionless frequencies of FG micro-plate with ref. [39]

a/h	l/h	**Method**	**Volume fraction index α**				
			0	1	2	5	10
5	0	Thai and Kim[39]	5.2813	4.0781	3.6805	3.3938	3.2514
		Present (FOSDT)	5.2854	4.0808	3.6817	3.3902	3.2517
	0.2	Thai and Kim[39]	5.7699	4.5094	4.0755	3.7327	3.5548
		Present (FOSDT)	5.7764	4.5139	4.0774	3.7275	3.5502
	0.4	Thai and Kim[39]	7.0330	5.6071	5.0763	4.5862	4.3200
		Present (FOSDT)	7.0446	5.6156	5.0799	4.5803	4.3097
	0.6	Thai and Kim[39]	8.7389	7.0662	6.4011	5.7137	5.3335
		Present (FOSDT)	8.7559	7.0789	6.4068	5.7089	5.3219
	0.8	Thai and Kim[39]	10.6766	8.7058	7.8861	6.9796	6.4759
		Present (FOSDT)	10.6987	8.7225	7.8938	6.9765	6.4650
	1	Thai and Kim[39]	12.7408	10.4397	9.4536	8.3193	7.6895
		Present (FOSDT)	12.7678	10.4604	9.4633	8.3177	7.6796
10	0	Thai and Kim[39]	5.7694	4.4192	4.0090	3.7682	3.6368
		Present (FOSDT)	5.7706	4.4200	4.0092	3.7667	3.6367
	0.2	Thai and Kim[39]	6.2537	4.8526	4.4006	4.0876	3.9162
		Present (FOSDT)	6.2556	4.8538	4.4010	4.0854	3.9141
	0.4	Thai and Kim[39]	7.5210	5.9664	5.4071	4.9169	4.6464
		Present (FOSDT)	7.5244	5.9689	5.4079	4.9143	4.6423
	0.6	Thai and Kim[39]	9.2543	7.4619	6.7580	6.0447	5.6487
		Present (FOSDT)	9.2592	7.4656	6.7593	6.0423	5.6440
	0.8	Thai and Kim[39]	11.2396	9.1537	8.2863	7.3338	6.8030
		Present (FOSDT)	11.2461	9.1586	8.2882	7.3319	6.7984
	1	Thai and Kim[39]	13.3651	10.9511	9.9101	8.7135	8.0448
		Present (FOSDT)	13.3731	10.9573	9.9126	8.7119	8.0404

The influences of thickness and length scale ratio on the vibration characteristics of the FG micro-plates with two piezoelectric layers are illustrated in Figure 4. It can be pointed out that by raising the length scale ratio, the stiffness of the micro-plate and then the natural frequency is increased.



(a) (b)

Fig 3. The natural frequency of the FG micro-plate with two piezoelectric layers for SSSS boundary condition versus thickness ratio for a) various length scale ratio (ℓ_c/ℓ_m) and b) various volume fraction index (α)

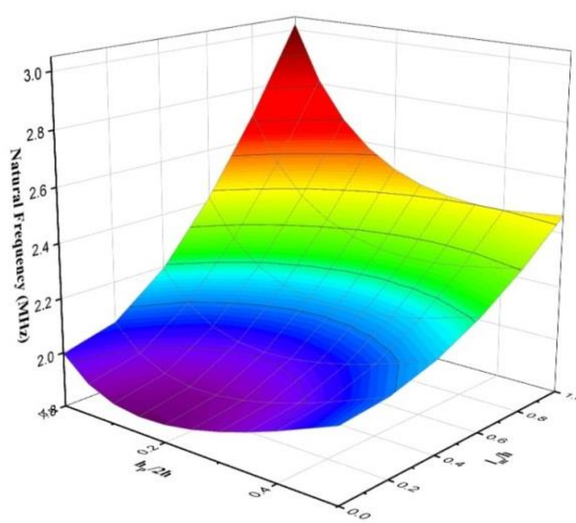


Fig 4. The impacts of thickness ratio ($h_p/2h$) and length scale ratio (ℓ_m/h) on the vibrating frequency of the FG micro-plate

The fundamental vibrating frequency for various boundary conditions is demonstrated in Figure 5. As observed, the more the rigidity of the boundary condition is, the more the stiffness of the micro-plate becomes. So, the natural frequency climbs by increasing the stiffness of the microstructure. As shown in Figure 5, the SSSS and CSCS boundary conditions have higher and lower natural frequencies, respectively.

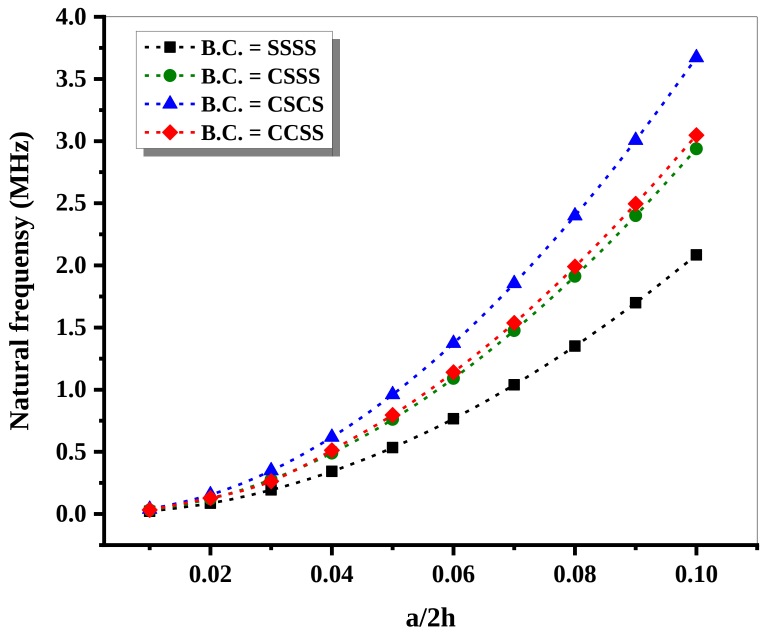


Fig 5. The fundamental resonance frequency of the FG micro-plate with two piezoelectric layers versus length to thickness ratio (a/2h) for various boundary conditions

8. Conclusion

In the current work, a theoretical solution possessing a variable length scale index is presented for the investigation of FG vibrating rectangular micro-plates with two piezoelectric layers. The differential equations related to the structure are formulated using Hamilton's principle based on the modified couple stress theory. The Galerkin method is utilized to discretize the derived differential equations. Also, the impacts of various variables such as geometrical ratios, boundary conditions, FG index, and length scale parameter on the resonance characteristics of the micro-plate are investigated. Moreover, it has been found that:

- The length scale ratio has a notable impact on the frequencies. The frequency is significantly raised as the length scale parameter inclines. This is because increasing the length scale parameter will make the micro-plate stiffer.

- By increasing the aspect ratio (a/b) and length to thickness ratio (a/2h), the frequency of FG micro-plates with two piezoelectric layers is increased. For a micro-plate with constant length a, the dimensionless frequency is increased by increasing the width or thickness.
- The volume fraction index has an inverse trend on the frequencies. Frequency gets lower values when the volume fraction index becomes greater.
- The thickness ratio (h_p/2h) has a noticeable impact on the vibration characteristics of FG micro-plates with two piezoelectric layers. By raising the thickness ratio, firstly the resonance frequency is decreased and then increased.
- The stiffness of micro-plates is prominently influenced by the boundary conditions. So, by increasing the stiffness, the natural frequency inclines.

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