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Investigation of Dynamic-Thermal Stress Intensity Factor in Functionally-Graded Plates Having an Edge Crack

Safa Peyman[*a], **Amir Eskandari**[b]

[a]*Assistant Professor, Department of Structural Engineering, Imam Hossein University, Tehran, Iran*

[b]*Ph.D. Student, School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran*

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Functionally graded materials (FGMs) are non-homogeneous materials whose properties gradually vary as a function of coordinates. Recently, due to the possibility of applying FGMs in conditions with severe temperature changes, e.g., nuclear reactors, chemical power plants, and space crafts, the interest in investigations on this type of material has increased considerably. Usually, FGMs are designed to tolerate drastic temperature changes and thermal shocks are mainly associated with thermal stresses. Therefore, the occurrence of thermal fracture in FGMs is probable. Consequently, studying the fracture mechanics of this type of material under extreme thermal shocks and dynamic loads has become crucial. This paper studies the first mode stress intensity factor (SIF) in FGM plates with an edge crack under thermal shock and dynamic loading. The plates consist of Nickel (metal) and Zirconia (ceramic) properties on top and bottom, respectively. The finite element method is employed to perform dynamic thermal analyses of the plates. Having known that variations in the amount of metal and ceramic used in producing an FGM plate can change its behavior, different gradients of material properties are applied for modeling the FGM plates. Then, the effects of the variation of the gradients on the dynamic-thermal SIF under thermal shocks and dynamic loadings are examined. One of the novelties of the present study is modeling the required material properties for thermal analyses (heat transfer coefficient, specific heat, and thermal expansion coefficient) and dynamic analyses (elasticity modulus, Poisson's ratio, and density) as functions, which have been rarely considered simultaneously in the previous studies.

[*] Corresponding author:
E-mail address: speyman@ihu.ac.ir (S. Peyman)

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1. Introduction

Nowadays, the use of layered materials, such as coated materials, in aerospace industries is significantly increasing. To compensate for the lack of materials that can withstand high amounts of residual and thermal stresses leading to cracking and discontinuity in the structure, functional materials (FGM) with continuous and gradual changes in thermal and mechanical properties are utilized as a suitable substitute for layer materials. Because the dominant failure mode of this type of material is fracture, the proper use of these materials depends on the correct understanding of their failure mechanics [1]. Linear elastic fracture mechanics is a method to express the field and stress distribution near the crack tip in terms of the load in the vicinity, the size, and the geometric shape of the crack or the discontinuity of the crack type. This method is based on applying the principles of linear elastic mechanics theory to the cracked part. The most influential principle of linear elastic fracture mechanics is the stress distribution near a pointed crack in terms of a quantity called the stress intensity factor (K) with a unit of $MPa.m^{0.5}$ (in terms of SI units), which is the amount of stress applied to the part and also the geometry Leave is dependent [2].

Determining dynamic stress intensity factors as important material failure parameters for understanding and predicting the behavior of a cracked object has always been an attractive research topic for researchers. Due to the application of functional materials and the importance of investigating failure in this type of materials, many researchers have studied the stress intensity factors in functional materials such as [3-8]. Considering the wide application of functional materials in various industries, analyzing the behavior of this type of material under the effect of dynamic loads becomes particularly important. In this regard, Chen [9] obtained the dynamic stress intensity factor of a rectangular sheet with a central crack using the finite difference method. By developing a particular component near the crack tip, Aoki and his colleagues [10] presented the relationship between the dynamic stress intensity factor and the displacement at the crack tip using the finite element method. Nishioka et al. [11] obtained the dynamic stress intensity factors of a homogeneous rectangular bar with a central oval crack using the finite element method. Qajar et al. [12] used the interactive integral to calculate the stress intensity factors under the effect of dynamic loading.

As mentioned earlier, one of the essential applications of functional materials is to use them in structures or parts exposed to thermal shock. Extreme temperature changes cause cracks in this type of material; therefore, for the correct use of these materials, it is necessary to conduct a sufficient study on the fracture mechanics of these materials under the effect of thermal shocks [13, 14]. This research investigates the stress intensity factor of the first mode in a beam with an edge crack made of functional materials (including two phases of metal and ceramic) under thermal and dynamic loading. The effect of different methods of changing the properties of materials on the stress intensity factor under the effect of each of the thermal, dynamic, and both loadings have been investigated. In the modeling of functional materials, it has been tried to model functionally all the characteristics that influence the behavior of materials under the effect of thermal-dynamic loading, and also the effect of inertia on the coefficient of dynamic-thermal stress has been investigated.

2. Stress intensity factor

As mentioned in the introduction section, stress intensity factors play an integral role in understanding the failure behavior of materials. To evaluate the stress intensity factors, various methods, such as displacement-based techniques (DCT) [10, 15] and the standard and modified integral J method [16, 17], are used. In this research, the first method is used, which is explained in the following. In the analysis of two-dimensional models, two deformation modes can be considered for cracks, which are the first mode or opening mode, which will bring the stress intensity factor K_I, and the second mode or shear mode, which will have the stress intensity factor K_{II} will bring Using relations 1 and 2, stress intensity factors can be calculated for the first and second modes, respectively, based on the displacement component of the points around the crack in the x-y plane [18].

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}cos\frac{\theta}{2}\left(K + 1 + 2\sin^2\frac{\theta}{2}\right) \tag{1}$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}sin\frac{\theta}{2}\left(K + 1 + 2\cos^2\frac{\theta}{2}\right) \tag{2}$$

$$u_x = \frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}sin\frac{\theta}{2}\left(K + 1 + 2\cos^2\frac{\theta}{2}\right) \tag{3}$$

$$u_y = \frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}cos\frac{\theta}{2}\left(K + 1 + 2\sin^2\frac{\theta}{2}\right) \tag{4}$$

In the above equations, r is the distance from the crack tip, u_x and u_y are the displacement components along the x and y axes, θ is the angle of crack expansion compared to the initial crack, μ is the shear modulus, and also K, if plane stress or plane strain, is equal to:

$$K = \begin{cases} \frac{3-\upsilon}{1+\upsilon} & (p.stress) \\ 3-4\upsilon & (p.strain) \end{cases} \tag{5}$$

It should be noted that the dynamic stress intensity factors are usually normalized to the value of the stress intensity factor in the static state Ks (see Equation 6). In Equation 6, σ_0 is the static stress and *a* is half of the crack dimension.

$$K_s = \sigma_0\sqrt{\pi a} \tag{6}$$

Calculation of Dynamic Stress Intensity Factors in Functional Materials

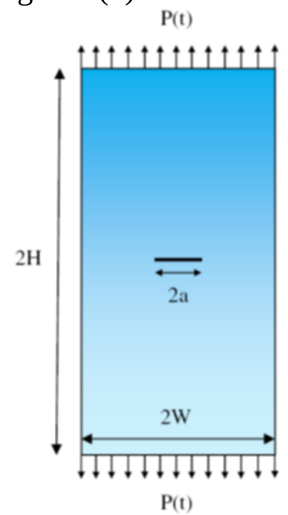
The direction of changes in the properties of the material of the functional plane studied in this research, whose characteristics will be explained more in the following sections, is such that only the first mode crack occurs. Therefore, to validate the method of calculating the stress intensity factor in functional materials under the effect of dynamic loading, the stress intensity factor of the first mode (K_I) for an interplane crack with variable functional materials in the vertical direction (for changes in the elastic properties of the material in the vertical direction) is in the direction of the crack, y) under the effect of dynamic loading has been calculated using Equation

1 and compared with the results of the investigation carried out on this model by Song [19]. Figure 1-a shows a plate modeled by Song [19] as plane strain. The length (2H) and width (2W) of the plate are 40 and 20 mm, respectively, and the crack dimension (2a) is equal to 4.8 mm. The modulus of elasticity (E) and density (ρ) are functionally modeled (Equations 7 and 8) and Poisson's ratio is considered constant, i.e., $\nu = 0.3$.

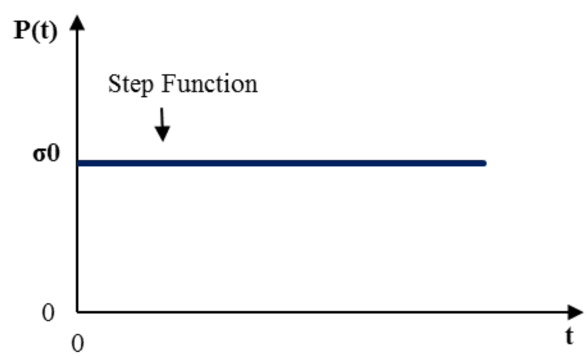
$$E = E_H \exp(\beta_1 y) \tag{7}$$

$$\rho = \rho_H \exp(\beta_1 y) \tag{8}$$

In the above relations, E_H and ρ_H are the modulus of elasticity and density of materials in a homogeneous state equal to 199.992 GPa, and 5000 Kg/m^3, respectively. Three values of 0, 0.05 and 0.1 have been considered for β_1, and this plate has been subjected to dynamic loading, as shown in Figure 1(b).



(a)



(b)

Fig. (1) (a) The considered plate for the calculation of dynamic stress intensity factors, (b) Dynamic loading

To validate the method of calculating the dynamic stress intensity factor in functional materials, the plate shown in Figure 1(a) has been modeled in ABAQUS software [20] and the UMAT subroutine has been used to apply the functional changes of the modulus of elasticity. In order to model the density as a function, considering that the UMAT subroutine is not able to consider the density, it is necessary to define the density as a function of an additional variable such as temperature. Other researchers have also used this method to consider density as a function; for instance, reference [21] can be mentioned. Therefore, the modulus of elasticity and density were considered functional based on Equations 7 and 8 (for three values of 0, 0.05, and 0.1 for β_1). Figure 2 shows the meshing of the modeled plate to verify the accuracy of the dynamic stress intensity factor calculation.

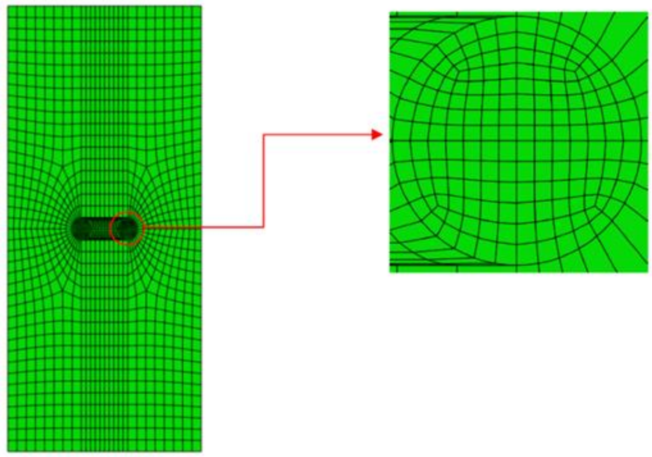
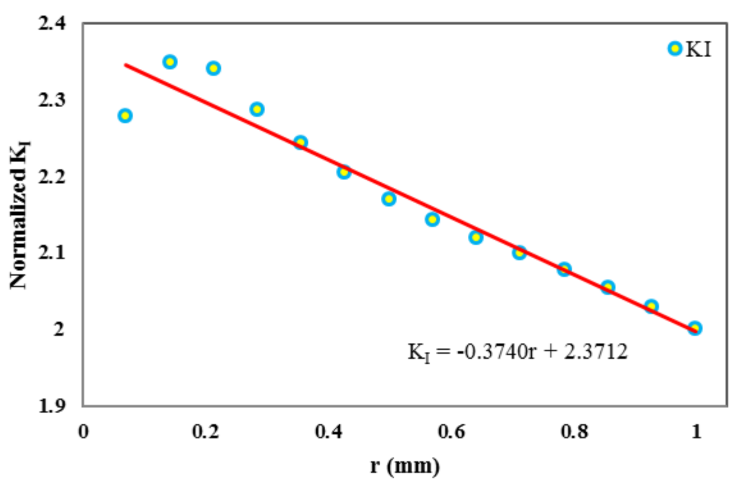


Fig. (2) Meshing of the modeled plate to verify the accuracy of calculating the dynamic stress intensity factor

Considering that in this research, the displacement method was applied to calculate the stress intensity factor, during the time of dynamic analysis, in each time step, for different values of the distance from the crack tip, r, different values of the stress intensity factors are obtained. By fitting a line to the values of the stress intensity factors along with their corresponding r values, the value of the stress intensity factor at the point r=0, which is the crack tip, can be calculated [21]. Figure 3 shows how to calculate the dynamic stress intensity factor for the intended crack in the modeled functional plane at an arbitrary time. As can be seen, the stress intensity factor for the first mode at this time is equal to the width from the origin of the regression line, which is the same coordinate as the fitted line at the crack tip (r=0). This operation is performed at each time step during the dynamic analysis.



1.

Fig.(3) Calculation of dynamic stress intensity factor of the first mode using displacement component

Figure 4 shows the time history of K_I for the crack in the desired functional plane under the effect of loading presented in Figure 1(b). As it is known, the obtained results are in good agreement with the results of Song [19], which shows the correctness of the modeling of the plate with functional materials as well as the calculation of the dynamic stress intensity factors. The time history of K_I was calculated for three values of β_1 and shown in terms of C_d and H. The C_d value of the longitudinal wave speed of homogeneous materials is equal to 7.34 mm/μsec.

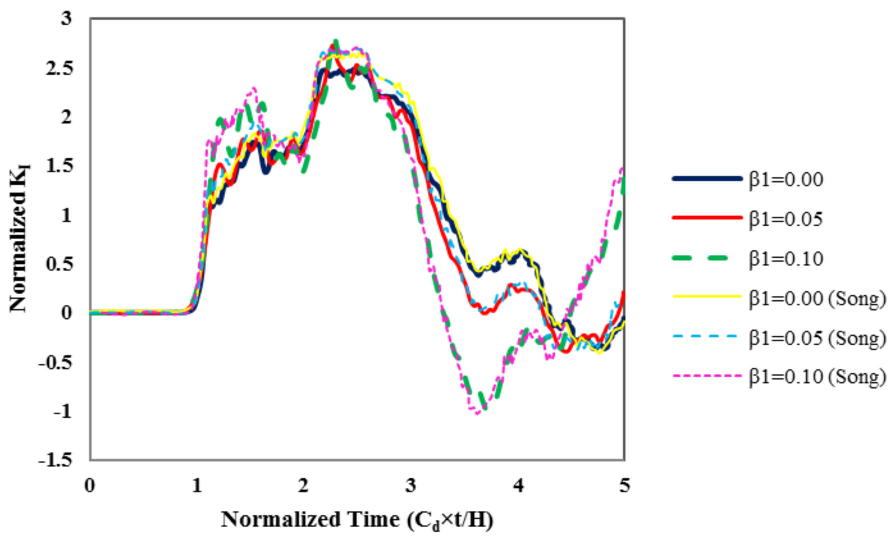


Fig.(4) Time history of stress intensity factor (K_I) under the effect of dynamic loading for different values of β_1

3. Validation of transient thermal analysis

In order to perform transient thermal analysis in FGMs, it is necessary to functionally define not only the modulus of elasticity, Poisson's ratio, and density but also other characteristics of materials, such as heat transfer coefficient, specific heat, and thermal expansion coefficient. In the previous section, it was mentioned that to define the modulus of elasticity and Poisson's ratio functionally, the UMAT subroutine was used in ABAQUS. It should be noted that density cannot be defined as a function of temperature to perform thermal analysis. Therefore, another method was used to model density, such as heat transfer coefficient (k), specific heat (c), and thermal expansion coefficient (α). In this method, the desired features are defined as a function of a field variable. It is obvious that to use this method, it is necessary to perform an additional analysis without any loading, and after creating the output file in which the desired field variable values are recorded, the desired characteristics can be defined in the new model based on the field variable recorded in the initial analysis. In this part, it is tried to validate this method of modeling FGMs for thermal analyses.

In order to verify the validity of the transient thermal analysis, the research conducted by Sutrahar et al. [22] on functional materials has been used. Figure 5 shows the cube considered for the thermal analysis of functional materials. The cube shown in the upper face is exposed to a thermal shock with a temperature of 100 degrees, and the lower face has a temperature of zero degrees. The following relations are considered to define the changes in heat transfer coefficient and specific heat as a function of z coordinates:

$$K(x, y, z) = k_0 e^{2\beta z} = 5e^{3z} \tag{9}$$

$$c(x, y, z) = c_0 e^{2\beta z} = 1e^{3z} \tag{10}$$

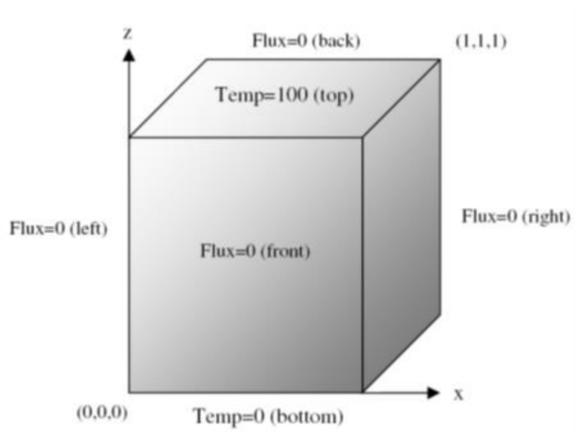


Fig.(5) The considered cube for validating transient thermal analysis of functional materials [22]

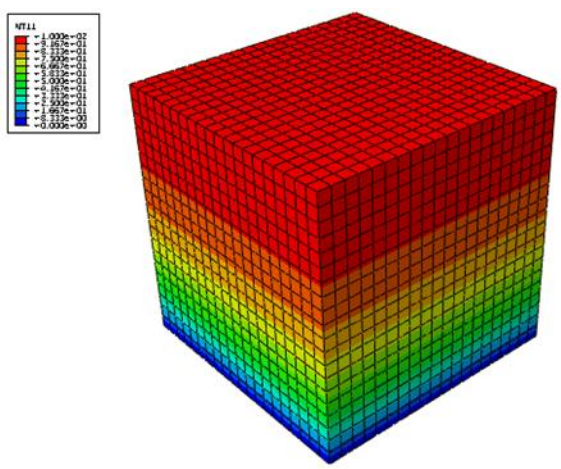


Fig.(6) Temperature contour of the cube's different points with functional materials considered in 0.5 sec

After performing the transient thermal analysis, the temperature of different points of the cube with the considered FGMs at 0.5 sec is depicted in Figure 6. Figure 7 compares the results obtained in this research (temperature of different points of the cube at different times) and the results obtained by Sutrahar et al. [22]. As can be seen, the results obtained from the thermal analysis in this study, which was performed using ABAQUS software [20], are in acceptable agreement with the results obtained by Sutrahar et al. [22]. Therefore, the correctness of conducting transient thermal analysis in functional materials is confirmed. It should be mentioned that to perform this analysis in ABAQUS software, 6-sided 8-node heat transfer elements were used and the heat transfer analysis type was selected.

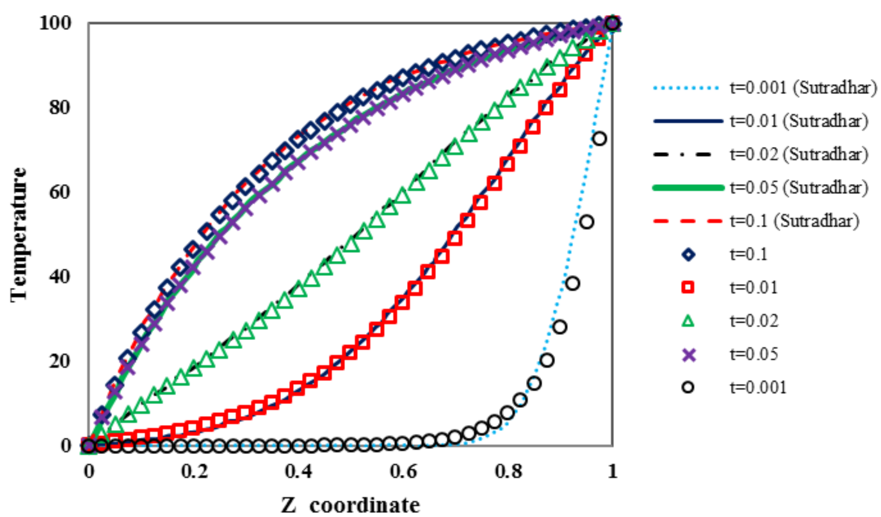


Fig.(7) Comparison of temperature distribution in the height of the cube with functional materials at different times in this study and the study conducted by Sutradhar et al. [22]

4. Intensity factor of dynamic thermal stress functional materials

In this part of the study, nickel and zirconia have been investigated as two metal and ceramic phases in functional materials. The mechanical and thermal characteristics of these materials are presented in Table 1. As can be seen, nickel has high hardness and heat transfer coefficient, and zirconia as ceramic has lower hardness and heat transfer coefficient. In fact, one of the main reasons for using functional materials in the form of a combination of metal and ceramic is that the desired structure or piece has metal properties on the side that is under the effect of impact or heavy loading and on the side, that is under the effect of severe changes. Thermal is supposed to have ceramic properties.

Table 1. Mechanical and thermal properties of nickel and zirconia

Material properties	Nickel-based super-alloy	Ceramic coating (ZrO_2-8% Y_2O_3)
Young's modulus, E (Pa)	2.2×10^{11}	5.00×10^{10}
Poisson's ratio, ν	0.3	0.25
Thermal expansion coefficient, α (K^{-1})	1.3×10^{-5}	9.0×10^{-6}
Density, ρ (kg m^{-3})	8900	4400
Thermal conductivity, K ($Wm^{-1}K^{-1}$)	91.2	2.94
Specific heat, Cp (J kg^{-1} K^{-1})	461	504

For the modeling of materials in ABAQUS software, the above-mentioned features are defined with compatible units as input for the software, and to check the different ratios of metal and ceramics, the changes in the ratio of materials are considered as exponents. The following equations present the general shape of material changes:

$$f_m = \frac{V_m}{V_m + V_c}, \quad f_c = \frac{V_c}{V_m + V_c} \tag{11}$$

$$P_{ef} = P_m f_f + P_c f_c \tag{12}$$

$$f_m = \left(\frac{y}{y_0}\right)^k, \quad f_c = 1 - f_m \tag{13}$$

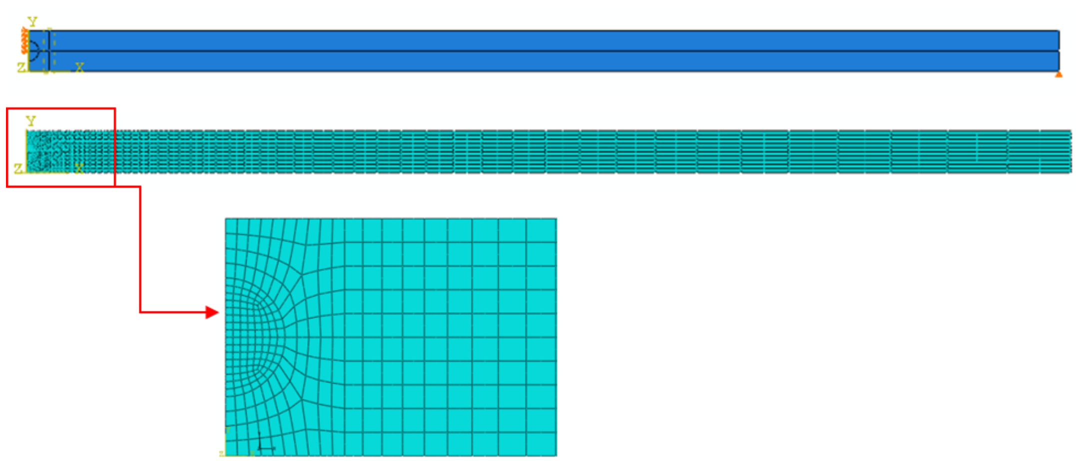


Fig. (8) Support conditions and meshing of the studied beam

In the above equations, f_m and f_c are the ratios of metal and ceramic in functional materials, respectively; and P_{ef} is one of the characteristics of functional materials (such as modulus of elasticity and density). The ratio of metal and ceramic depends on the k parameter. In order to study the effect of thermal shock on the dynamic stress intensity factor, a beam with simple supports with dimensions of 600 and 10 mm, in the middle of which there is an edge crack with a length of 5 mm, has been investigated in plane strain mode. Is. Due to the symmetry with respect to the y-axis (vertical axis), half of this beam is considered in the modeling. Figure 8 illustrates the support conditions as well as the meshing of the model (for modeling half of the beam) under study.

To investigate the influence of thermal shock and also dynamic loading on the stress intensity factor of the first mode, the lower side of the aforementioned beam has been subjected to a thermal shock of -100 degrees and dynamic load in the form of an impact with a constant size of 1 Newton during the analysis and the crack has entered the upper side of the beam. In modeling this beam, the materials are considered homogeneously and functionally. Homogeneous materials in the form of metal and ceramics have been used according to the specifications in Table 1. Equation 9 has been used for different values of k to check functional materials. In the lower part (y=0), which is exposed to thermal shock, the desired beam has ceramic characteristics and in the upper part (y=10), which is subjected to dynamic load, it has metal properties.

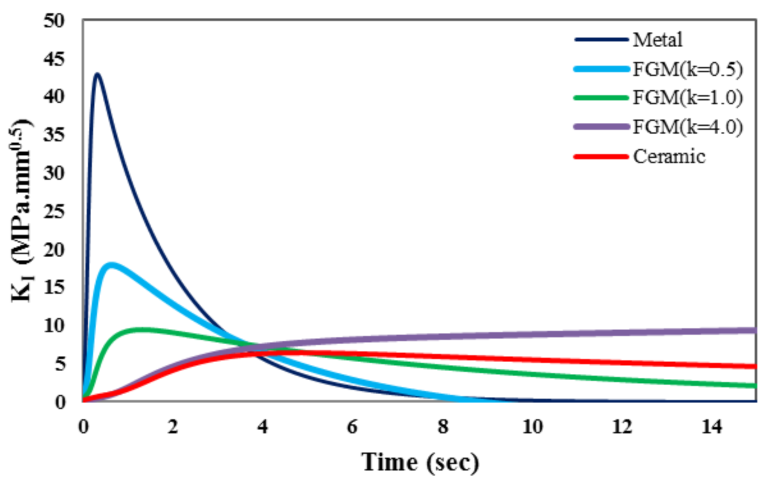


Fig. (9) The history of the stress intensity factor under the effect of thermal shock when the effect of inertia is ignored (for different materials)

Figure 9 represents the history of the stress intensity factor under the effect of thermal shock (temperature reduction in the lower face) for different materials, which was obtained using the heat-displacement coupling analysis [20]. It should be noted that in this type of analysis, the effect of inertia is ignored. As can be seen, the highest value of the stress intensity factor is related to the case where metal is used homogeneously, and the lowest value is related to the case where ceramic is used homogeneously. Another point that can be inferred from the figure is that with the increase in the amount of metal in functional materials due to the higher heat transfer coefficient of metal compared to ceramic, the maximum value of the stress intensity factor is observed in a shorter period of time.

In Figure 10, the effect of dynamic loading (fixed load of 1 Newton) alone on the coefficient of stress intensity in homogeneous materials (metal and ceramic) and functional materials with different gradients of changing material properties has been investigated. As can be seen, under the effect of dynamic loading, the lowest value of the stress intensity factor is related to the case where functional materials are used (assuming k=4). In other words, the use of functional materials for k=4 between two metal and ceramic phases has caused a significant reduction in the value of the dynamic stress intensity factor.

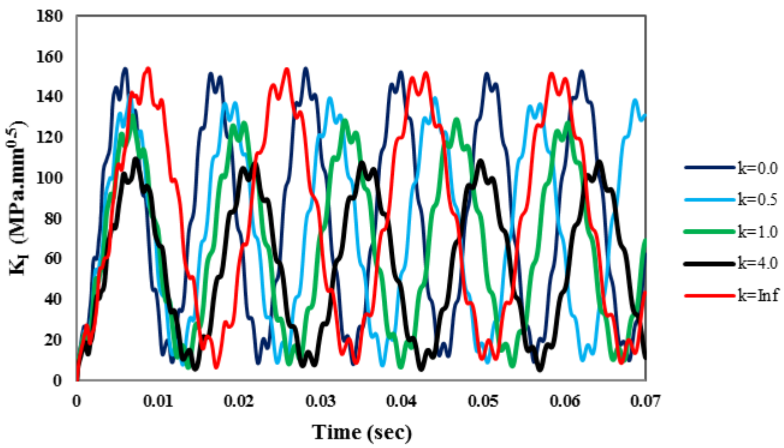


Fig. (10) Time histories of dynamic stress intensity factor in homogeneous and functional materials (under the effect of dynamic load)

If the stress-displacement solution is dependent on the temperature field, but on the contrary, the temperature field is not dependent on the stress-displacement solution, a sequentially coupled thermal-stress analysis [20] can be used. This analysis is performed in such a way that a heat transfer analysis is performed first, and then the temperature values obtained at each point over time are used as a predetermined temperature field for implicit dynamic stress analysis. It should be noted that by using the heat-displacement sequential coupling analysis, the effect of inertia can also be taken into account.

In this part of the study, an attempt has been made to investigate the effect of thermal shock separately, as well as thermal shock and dynamic impact simultaneously, on the stress intensity factor by using sequential heat-displacement coupling analysis. Implicit dynamic analysis has been used to consider the effect of dynamic impact separately. In Figure 11, the desired beam in three states of metal (k=0), ceramic (k=Inf) and also functional materials (k=4) is only subjected to thermal shock (without applying dynamic load). It should be noted that this result corresponds to the period of 0.28 seconds of the analysis time. As can be seen, the use of metal materials causes the highest value of dynamic-thermal stress intensity factor. The fluctuations observed in the history of stress intensity factor with constant amplitude show the effect of inertia. Figure 11(a) also shows the history of the dynamic-thermal stress intensity factor in the case where the inertia effect is ignored. As can be seen, in the given problem, inertia has a significant effect on the dynamic-thermal stress intensity factor, and ignoring this effect brings a significant error. This effect has been omitted some research in the technical literature [24-26]. As seen in Figure

11, under the effect of thermal loading, the lowest fluctuation value of the stress intensity factor due to inertia is related to the state in which functional materials are used (Figure 11(b)).

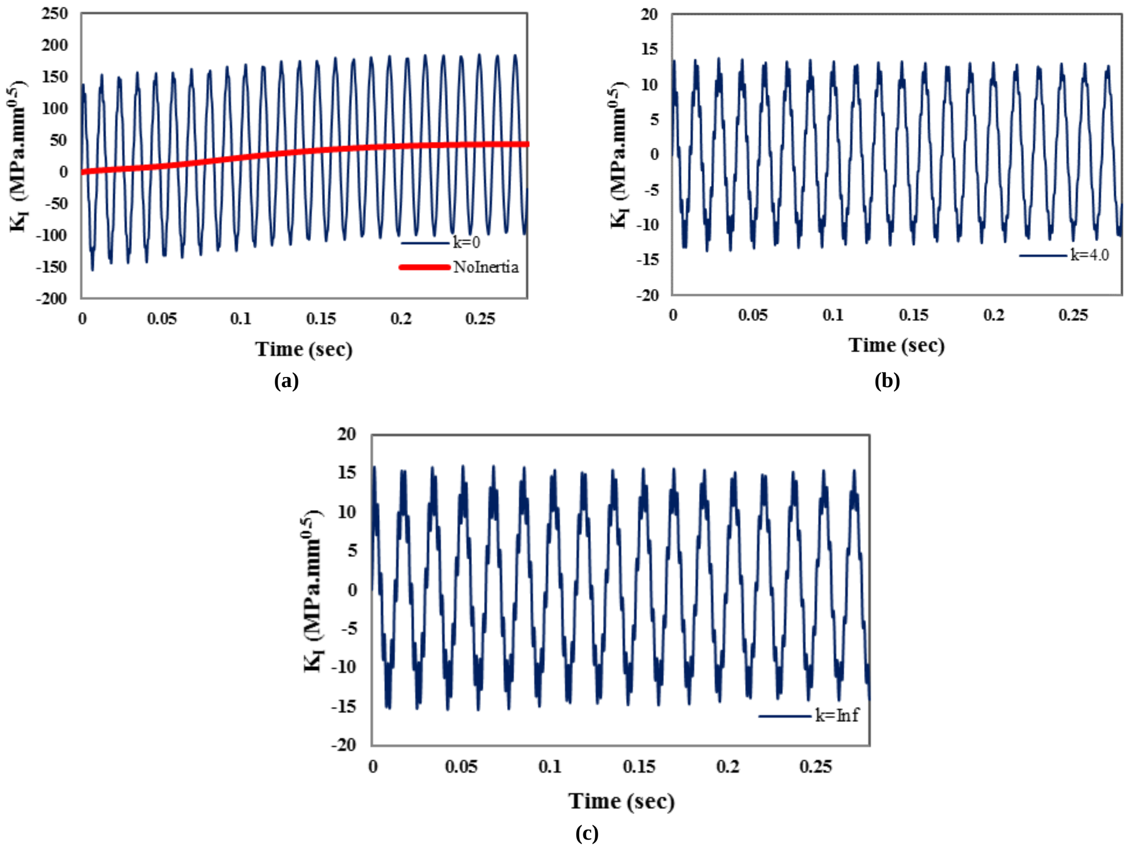


Fig. (11) Time history of dynamic-thermal stress intensity factor of the studied beam (for three different values of k) under the effect of thermal shock of -100 degrees

Figure 12 shows the time histories of the dynamic-thermal stress intensity factor of the desired beam under the simultaneous effect of thermal shock (-100 degrees on the lower face) and dynamic load (with a constant value of 1 N). As can be seen, in this case, the lowest value of the stress intensity factor is related to the case where functional materials are used. Figure 13 Time histories of the dynamic-thermal stress intensity factor for the functional plane (k=4) which, in addition to dynamic loading (impact) on two faces (lower face and crack edge), was subjected to thermal shock, with a temperature decrease of 100 degrees. In this figure, the results related to two cases of considering inertia and ignoring inertia are shown. As can be seen in the figure, the effect of inertia is significant. In some research works, the impact of inertia is ignored [24-26], but it should be noted that if this effect is ignored without full knowledge of the problem conditions, a considerable error may be created in the results. In this figure, the curve obtained by ignoring the effect of inertia accurately shows the general trend of the dynamic-thermal stress intensity factor, but it does not show the maximum or minimum value accurately. For future research works, SIFs for cracks under thermal impact based on different theories (e.g., Green-Naghdi theory [27]) or models (e.g., Lord-Shulman model [28,29]) can be investigated.

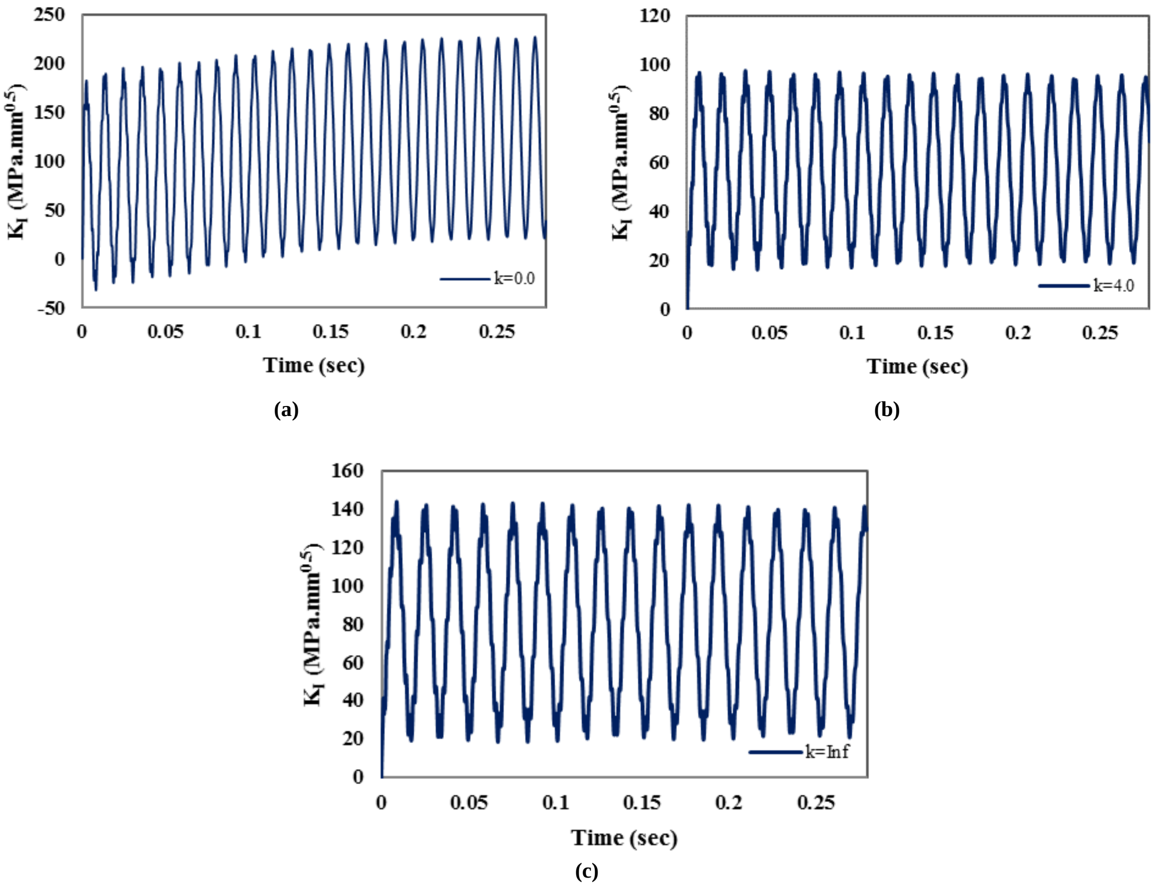


Fig. (12) Time histories of dynamic-thermal stress intensity factor under the effect of thermal-dynamic loading (for three different values of k)

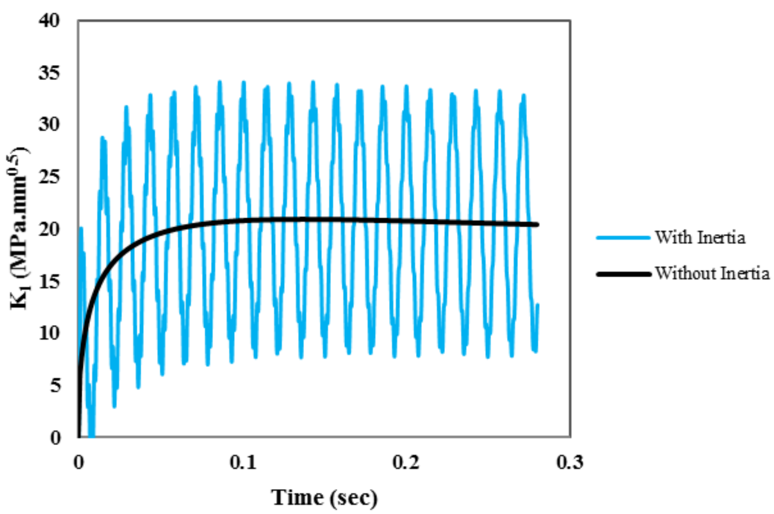


Fig.(13) The time history of the dynamic-thermal stress intensity factor in the desired beam under the effect of dynamic loading and the thermal shock that enters the lower side and the edge of the crack.

5. Conclusion

In this research, to investigate the influence of thermal shock and dynamic loading on the stress intensity factor in functional materials, the method of modeling functional materials and calculating the dynamic stress intensity factor was first validated. FGMs, including two phases of metal (nickel) and ceramic (zirconia) with different gradients of changing material properties, were considered for beam modeling. Examining the dynamic-thermal stress intensity factor under the effect of thermal shock independently and ignoring the inertial effect (using heat-displacement coupling analysis) revealed that the highest value of the stress intensity factor corresponds to the case where the metal is used homogeneously. Likewise, the lowest value corresponds to the case where ceramic is used homogeneously. It was also demonstrated that by increasing the amount of metal in FGMs due to the higher heat transfer coefficient of metal compared to ceramic, the maximum value of the stress intensity factor is observed in a shorter period of time. The result of examining the stress intensity factor just under the effect of dynamic loading (fixed load of 1 N) in homogeneous materials (metal and ceramic) and functional materials with different gradients showed that the lowest value of the stress intensity factor is related to the state of functional materials (for k=4). In examining the dynamic-thermal stress intensity factor under the effect of thermal shock (-100 degrees on the lower face), fluctuations in the history of the stress intensity factor with constant amplitude were observed, which represents the significant effect of inertia, and it was found its ignorance leads to considerable error. It was indicated that the lowest value of the stress intensity factor corresponds to the state where functional materials are employed. It should be noted that the results obtained in this research are related to the materials considered and also the type of applied loading, and in general, it indicates that for the use of FGMs under the effect of thermal shock and dynamic loading, it is possible to use an optimal gradient to change the properties of the materials so that the value of the stress intensity factor is minimal. However, this optimal gradient definitely depends on the type of thermal and dynamic loading.

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