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Vibration analysis of two-layered microplate based on the modified couple stress theory using penalty approach

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ABSTRACT

Predicting the vibration behavior of microsystems is of great importance. In this study, the vibration behavior of a microsensor modeled as a two-layer microplate is investigated. The effect of size has been investigated through the modified couple stress theory. The first natural frequency is extracted using the penalty approach. Boundary conditions are modeled using linear or torsional springs. Finally, changes in the natural frequency of the microsystem are presented according to different values of the microplate parameters such as the thickness of the silicon layer and material of the second layer. The results show that the natural frequency decreases as the thickness of the second layer increases. In addition, despite the different first natural frequencies for different parameters, the natural frequency diagram shows the same behavior in terms of system parameters under various boundary conditions. Finally, the effect of the thicknesses ratio h_2/h_1 and material length scale parameters ratio l_2/l_1 on the natural frequency is investigated.

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1. Introduction

Microelectromechanical systems (MEMS) have advanced greatly due to their small size, lightweight, high performance, easy mass production and low cost. Microelectromechanical sensors are used to measure mechanical variables such as displacement, velocity, acceleration and force [1]. Microbeams and microplates are used in many MEMs sensors. Microsensors are widely used to measure some mechanical parameters and many of them consist of two major layers, piezoelectric and silicon. In many mems devices, the change of the special parameter is related to the change of the natural frequency. For example, in a microsensor for virus detection, when the viruses sit on the microsensor area, the natural frequency decreases. These devices can be modeled as a microplate or microbeam [2]. Many researchers have studied the dynamic characteristics of microstructures. Jomehzadeh et al. [3] analyzed microplate vibration using a new model based on the modified couple stress theory (MCST). The new model proposed in [3] can explain the effect of scale on microplate vibration. The equations of motion were obtained based on Kirschhoff's hypothesis using Hamilton's principle. Liao-Liang Ke et al.[4] developed a microplate-free vibration based on the Mindlin plate theory and the modified coupled stress theory. Hamilton's principle is used to derive the governing equation and boundary conditions in their paper. They used the p-version Ritz method to determine the natural frequency of the microplate with different boundary conditions. Ramezani [5] investigated the nonlinear vibration of the Kirchhoff a microplate based on strain gradient theory. In this model, the Von Carman strain tensor is used to include geometric nonlinear effects. The ordinary differential equation for the first mode of nonlinear vibration was used for a simple microplate using the Galerkin method. Ansari et al. [6] investigated the bending, buckling and free vibration behavior of the FG circular microplate based on the modified strain gradient elasticity theory and the Mindlin plate. In [6], the discretization of the governing differential equations with different boundary conditions is carried out with the generalized differential quadrature (GDQ) method. Şimşeket al.[7] studied the vibrations of a microplate with a moving load. They used the modified couple stress theory to investigate the vibrational behavior of the microplate. The dynamic response of the microplate has been investigated by considering the effects of the material length scale parameter on the plate dimensions, boundary conditions and moving load speed. The effect of dynamic deviations from the parameter of load size and speed is one of the results of his study. Mirsalehi et al.[8] studied the free vibrations of an FGM microplate based on strain gradient theory. The results of their study indicate that increasing the length scale parameter increases the critical buckling load and vibration frequency similar to the macroscopic model. Şimşek et al.[9] investigated the static bending and forced vibration of a functionally calibrated microplate under moving load using modified coupling stress theory. They used Lagrange equations to obtain microplane motion equations. The influence of the scale parameter and the moving load velocity on the dynamic response has been investigated. Omiddezyani et al. [10] studied the vibration of a rectangular microplate. One side of the microplate is coupled with an ideal fluid. MSCT was used in mathematical modeling of the problem. They used the Hamilton principle to derive equations and boundary conditions. The problem of the obtained eigenvalue related to the free vibration of a rectangular microplate with simple supported with fluid was solved analytically using the Rayleigh-Ritz method. Thai et al. [11] proposed a size-dependent computational model for the free vibration of multilayer functionally graded GPLRC microplates. This model is based on a modified strain gradient theory and higher-order shear deformation theory. They used the principle of virtual work in deriving the equations. Zhou et al. [12] developed a circular microplane with surface effects and investigated

the effect of surface effects and residual stresses on dynamic properties. They derived the approximate results of their study using the Galerkin method. Thai et al.[13] examined the free vibration of the FG hexagonal beryllium crystal microplate. The natural frequency of the microplate was extracted based on the ISO geometric analysis (IGA). Their astrological results show that the frequency of the dependent model is larger than the frequency of the classical model. Mohammad-Rezaei Bidgoli and Arefi [14] studied the free vibration of composite microplate reinforced with functionally graded Nanoplatelets based on modified strain gradient theory. They calculated the size dependence in the governing equations with three parameters of the material length scale. The Hamilton principle is used to derive the equations. To validate their study, they reduced their relationships to the modified couple stress theory and the modified strain-gradient theory. Rahi [15] investigated a vibrational behavior of a two-layer micro sensor using the modified couple stress theory. He assumed the microsensor to be a microbeam. He used the Hamilton principle to derive the governing equations. The results of his research show that the first natural frequency of a microsensor decreases with increasing the scale parameter of dimensionless material or decreasing the thickness of silicon and piezoelectric. Rahi [16] investigated the dynamic response of multilayer microbeams. He derived the governing equations using Hamilton's principle and analytically extracted the natural frequencies of the system. Ansari et.al [17] investigated Size-dependent nonlinear bending and post-buckling of functionally graded Mindlin rectangular microplates considering the physical neutral plane position. They used MCST, the power law function, and the Hamilton principle to derive the equations. The nonlinear bending and post-buckling responses of FG microplates were investigated by considering the effects of material gradient index, length scale parameter, length-to-thickness ratio and boundary conditions. Thanh et.al [18] studied size-dependent thermal bending and buckling composite laminate microplates. They used a new modified couple stress theory and isogeometric analysis to analyze the model. Ansari et.al [19] investigated the forced vibration of functionally graded non-classical microplates. The microplates are subjected to a transverse harmonic excitation force. They used the Hamilton principle to derive governing equations and the Galerkin method to convert them to a set of ordinary differential equations. The frequency response curve and effects of some parameters such as material types were obtained. Gholami et al. [20] investigated the nonlinear pull-in instability and vibrations of electrostatic actuators made of nanocrystalline materials under the influence of grain size and nanoscale effects. Governing equations are derived in the discretized weak form using the variational differential quadrature (VDQ) method based on the third-order shear deformation beam theory. The influences of various factors such as length scale parameter and density ratio on the pull-in instability and free vibration were investigated. Kumar et.al [21] analyzed thermoelastic damping for size-dependent microplate resonators. They used MCST and considered plane stress conditions and the three-phase-lag heat conduction model. The variations of thermoelastic damping as functions of the normalized frequency, microplate thickness, and length-scale parameter were investigated.

In this paper, the vibrational behavior of a two-layer microsensor is investigated. For microsensor analysis, the system is modeled as a two-layer microplate and Non-classical Kirchhoff – Love plate theory is used. Modified couple stress theory is used to investigate the effect of the size effect parameter. Finally, natural frequencies are extracted using the Rayleigh-Ritz method for two different boundary conditions. For the first time, the governing equations of the two-layer microplate are derived using the penalty approach and the boundary conditions are modeled as linear or torsional springs.

2. Modeling and governing equations of the system

The location of the neutral surface relative to the middle surface in FGM materials can be expressed as follows [9]:

$$e = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} E(z_m)dz_m}{\int_{-\frac{h}{2}}^{\frac{h}{2}} E dz_m} \tag{1}$$

The z_m is the transverse coordinates defined with respect to the middle surface of the cantilever microplate. Rahi uses Eq. 1 for a two-layer beam to express this distance as follows [15]:

$$e = \frac{1}{2}\frac{(E_2 - E_1)(h_2^2 - hh_2)}{E_1 h + (E_2 - E_1)h_2} \tag{2}$$

Where, E_1 and E_2 are the elastic modulus of the first and second layers, respectively. The strain energy of an isotropic linear elastic body can be calculated based on the modified couple stress theory in Eq. 3 [15]:

$$\pi_s = \frac{1}{2}\int(\sigma_{ij}\,\varepsilon_{ij} + m_{ij}\chi_{ij}^s)\mathrm{dv} \;\; ; \;\; \mathrm{i=x,\,y,\,z} \tag{3}$$

Where π_s is the strain energy, σ_{ij} and ε_{ij} show stress and strain tensors, respectively. The second term on the right-hand side of Eq. 3 ($m_{ij}\chi_{ij}^s$) is related to the MSCT, in which m_{ij} shows the components of the deviatoric part of the couple stress tensor and the χ_{ij}^s components of the symmetric curvature tensor. For a two-layer plate, the appropriate form of Eq.3 is

$$\begin{aligned}\pi_s = \frac{1}{2}\int_0^a\int_0^b\int_{\frac{h}{2}-e-h_1}^{\frac{h}{2}-e}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}^s\right)dzdydx \\ + \int_0^a\int_0^b\int_{-\frac{h}{2}-e}^{\frac{h}{2}-e-h_1}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}^s\right)dzdydx = \int_0^a\int_0^b[(k_1\alpha_1 + k_2\alpha_2 \\ + \frac{\beta_{1+\beta_2}}{2})\left[\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w_0}{\partial y^2}\right)^2\right] + 2\left(\nu_1 k_1\alpha_1 + \nu_2 k_2\alpha_2 + \frac{\beta_{1+\beta_2}}{2}\right)\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} \\ + (\frac{k_1\alpha_1(1-\nu_1) + k_2\alpha_2(1-\nu_2)}{2} + 2(\beta_1 + \beta_2))\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2]dydx\end{aligned} \tag{4}$$

The strain and stress tensors can be expressed as follows [15]:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \frac{E_n}{1-\nu_n}\begin{bmatrix}1 & \nu_n & 0\\ \nu_n & 1 & 0\\ 0 & 0 & \frac{1-\nu_n}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix} \tag{6}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{7}$$

$$(m_{ij})_n = \frac{E_n^2 L^2}{1-\nu_n}\chi_{ij}^s \tag{8}$$

In the above equations ϵ_{ijk} and δ_{ij} denote the permutation tensor and Kronecker delta respectively. u_i is the component of the displacement vector. L and ν are the material length scale parameter and Poisson's ratio respectively. β, α, k_1 and k_2 are defined as follows:

$$(\beta)_n = \frac{E_n^2}{1-\nu_n}\ ,\ (\alpha)_n = \frac{E_n}{1-\nu_n}$$
$$k_1 = \frac{1}{3}\left(\left(\frac{h}{2}-e\right)^3 - \left(\frac{h}{2}-e-h_1\right)^3\right) \tag{9}$$
$$k_2 = \frac{1}{3}\left[\left(\frac{h}{2}-e-h_1\right)^3 - \left(-\frac{h}{2}-e\right)^3\right]$$

The two-layer microplate with length a, width b, and thickness of h_1 and h_2 is shown in Figure 1.

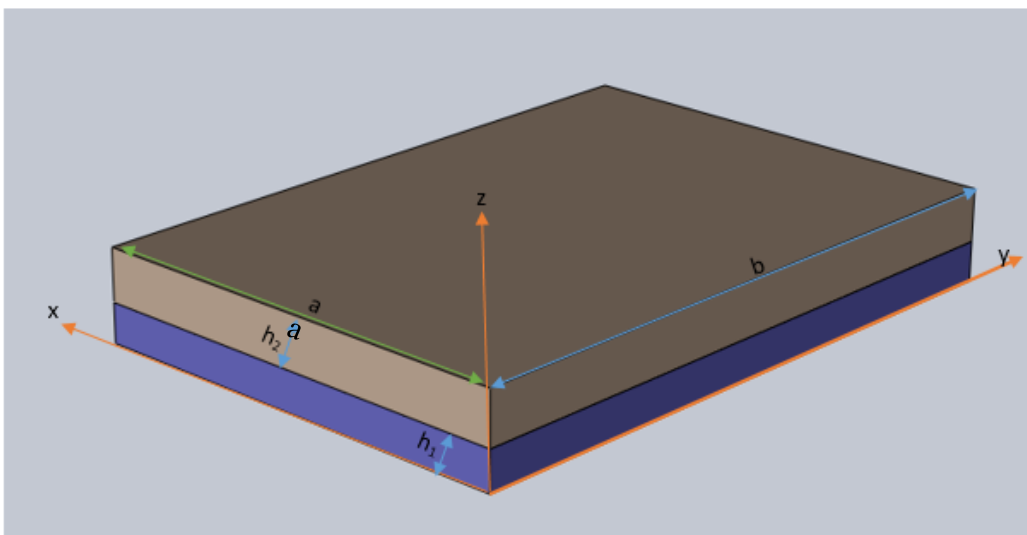

Fig. (1) Two-layer microplate

According to Kirchhoff–Love microplate theory, displacement components can be written as follows [7]:

$$u_x(x,y,z,t) = -z_n \frac{\partial w_0(x,y,t)}{\partial x} \tag{10}$$

$$u_y(x,y,z,t) = -z_n \frac{\partial w_0(x,y,t)}{\partial y} \tag{11}$$

$$u_z(x,y,z,t) = w_0(x,y,t) \tag{12}$$

Displacements in the directions of X, Y, and Z are expressed by u_x, u_y and u_z respectively. w_0 shows the transverse displacement and t denotes the time. The following strains are produced from the kinematic relations as follows:

$$\varepsilon_{xx} = -z_n \frac{\partial^2 w_0}{\partial x^2}\ , \varepsilon_{yy} = -z_n \frac{\partial^2 w_0}{\partial y^2}, \varepsilon_{xy} = -z_n \frac{\partial^2 w_0}{\partial x \partial y} \tag{13}$$

where ε_{xx} and ε_{yy} are the normal strain and ε_{xy} is the shear strain. By placing Eq. 9 and 10 in Eq. 6, the rotation components are calculated as follows:

$$\theta_x = -z_n \frac{\partial w_0}{\partial y}, \theta_y = -z_n \frac{\partial w_0}{\partial x}, \theta_z = 0 \tag{14}$$

Placing Eq. 13 in Eq. 6 results in determining the non-zero components of curvature as follows:

$$\chi_{xx} = \frac{\partial^2 w_0}{\partial x \partial y}\ ,\ \chi_{yy} = -\frac{\partial^2 w_0}{\partial x \partial y},\ \chi_{xy} = \frac{1}{2}\left(\frac{\partial^2 w_0}{\partial y^2} - \frac{\partial^2 w_0}{\partial x^2}\right)$$
$$\chi_{xz} = \chi_{yz} = \chi_{zz} = 0 \tag{15}$$

The kinetic energy of the system is expressed as follows:

$$T = \frac{1}{2}\int_0^a \int_0^b (\rho_1 h_1 + \rho_2 h_2)\,(\frac{\partial w_0}{\partial t})^2 dydx \tag{16}$$

3. Penalty approach

The penalty approach is used to obtain the natural frequency of the microplate. This method can be used to solve any equation regardless of boundary conditions. In the first step, it is assumed that there is no limitation for boundaries and the plate is on the FFFF condition (all edges are free). Three admissible functions are considered for each direction. These functions can model the displacement of the plate [22]. They are defined as:

$$\varphi_i(x) = 1 for i = 1\ ;\quad \psi_i(y) = 1 for i = 1$$
$$\varphi_i(x) = \left(\frac{x}{L}\right) for i = 2\ ;\ \ \psi_i(y) = \left(\frac{y}{L}\right) for\ i = 2 \tag{17}$$
$$\varphi_i(x) = (\frac{x}{L})^2\ \ for\ i = 3\ ;\ \ \psi_i(y) = (\frac{y}{L})^2\ \ for\ i = 3$$

In this method, boundary conditions are modeled by linear and torsional springs with high stiffness to express displacement and rotational limitations respectively. Based on Rayleigh-Ritz method, these springs change the potential energy, the stiffness matrix and thus the system's natural frequency. In this article, linear and torsional springs are called k_l and k_t. The potential energy of springs is defined as:

$$\pi_{k_l} = \frac{1}{2} w(x,y)^2$$
$$\pi_{k_t} = \frac{1}{2}\left(\frac{\partial w}{\partial R}\right)^2\quad R = x\ , y \tag{18}$$

The value of k_l and k_t are 10^p. Also, p is a positive integer. The value of p starting from 1 is increased by 1 unit in each step until the results converge. Figure 2 shows a convergence in natural frequency of a two-layers SSSS microplate with these characteristics:

Table 1. Characteristics of instance microplate

Layer number	material	E(GPa)	ν	ρ(Kg/m3)	h(μm)	a(μm)	b(μm)
1	Si	170	0.22	2233	8	600	600
2	PZT	63	0.32	7750	8	600	600

Based on the Rayleigh-Ritz method, natural frequencies are calculated as:

$$(\pi_t)_{max} = (\omega T^*)_{max} \tag{19}$$

Where

$$\pi_t = \pi_s + \pi_{k_l} + \pi_{k_t}$$
$$\omega T^* = T \tag{20}$$

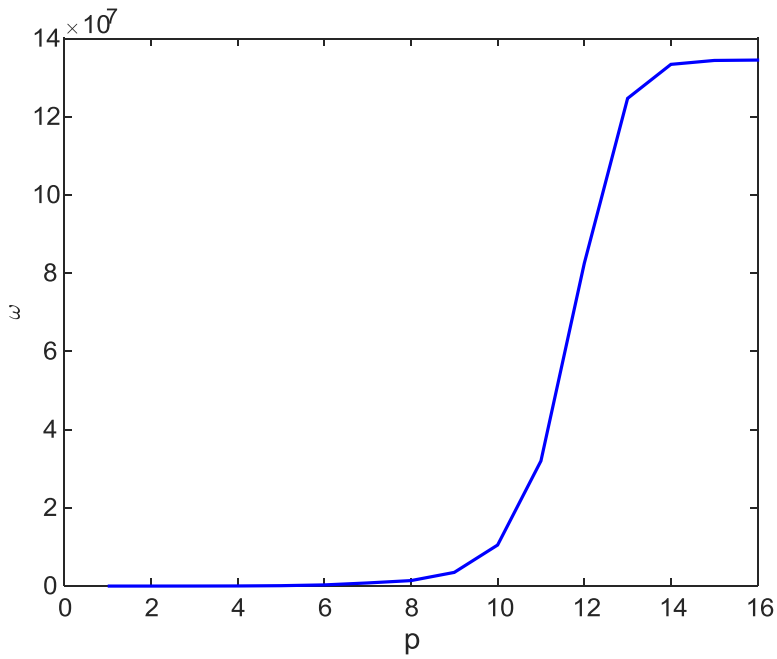

Fig. (2) Results from the convergence table for instance microplate

4. Results and discussion

In this section, the effect of two parameters, 2nd layer thickness and 2nd layer material, for a two-layer microsensor whose layer specifications are described in Tables 3 and 4 is examined. To investigate the two parameters expressed, the MATLAB code of the Rayleigh-Ritz method is written under two common boundary conditions in microsensors, which are CFCF (2 edges are free and 2 edges are clamped) and the second one is CFFF (1 edge is clamped and others are free).

Table 3. Characteristics of layers

material	E(GPa)	ν	ρ(Kg/m^3)
1st layer-silicon	170	0.22	2233
2nd layer PZT	63	0.32	7750

Figures 3 and 4 show the variation of the first natural frequency with the (a/h) ratio, where a=b, for three different values of the thickness ratio (h_2/h_1). Results show that when (a/h) increases, the natural frequency of the microplate decreases for both cases. Also in the high (a/h) ratio, the effect of the thickness ratio on the natural frequency, compared to low ratios, decreases. As can be seen, the frequency change does not depend on the boundary conditions.

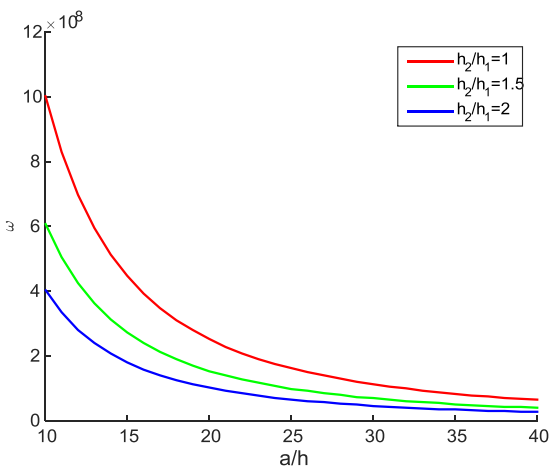

Fig. (3) Natural frequencies of CFFF microplate

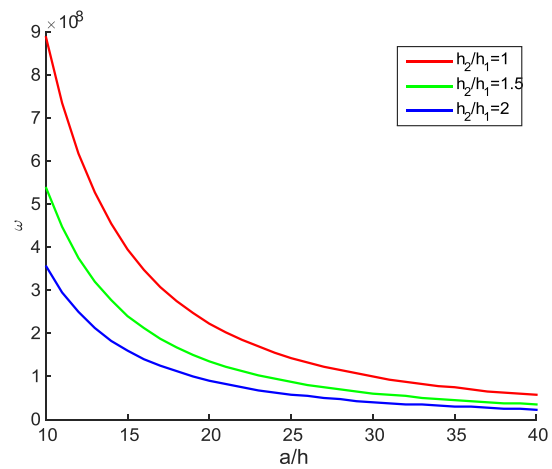

Fig. (4) Natural frequencies of CFCF microplate

The effect of the 2nd layer's material is investigated and the results are illustrated in Figures 5 and 6 for CFFF and CFCF cases respectively. As can be seen, the natural frequency decreases as the thickness of the second layer increases. The results also show that the natural frequency decreases with decreasing modulus of elasticity and has almost the same behavior in different materials.

Table 4. Characteristics of the second layer

Material	**E** (GPa)	ν	ρ (Kg/m^3)
AlN	308.3	0.179	3260
ZnO	112.2	0.336	5530
PZT	63	0.32	7750

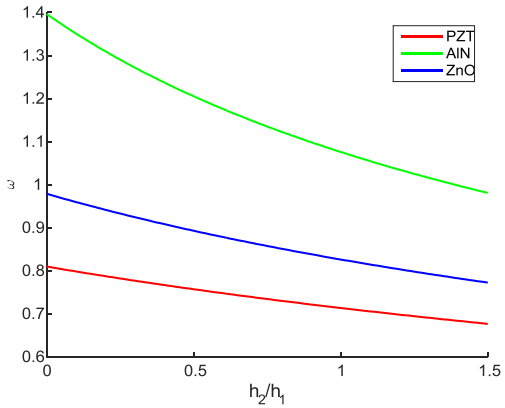

Fig. (5) Natural frequencies of CFFF microplate with a different second layer

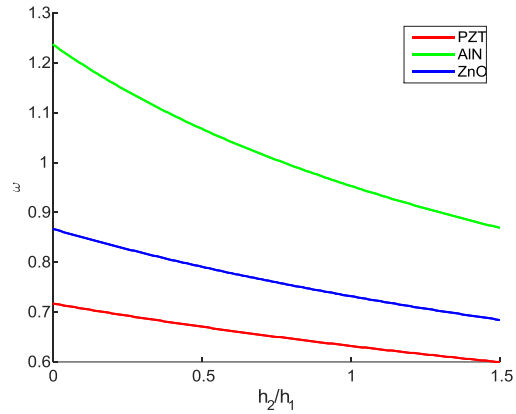

Fig. (6) Natural frequencies of CFCF microplate with a different second layer

In the last section of this study, the effects of material length scale parameters ratio (MLSPR) are investigated. Figure 7 shows the changes in the natural frequency based on the various MLSPR for plates with different dimensions and boundary conditions. The natural frequency of the microplate increases when the MLSPR increases. This increase is high as the thickness of the plates decreases. Also, there is the same behavior in two types of boundary conditions.

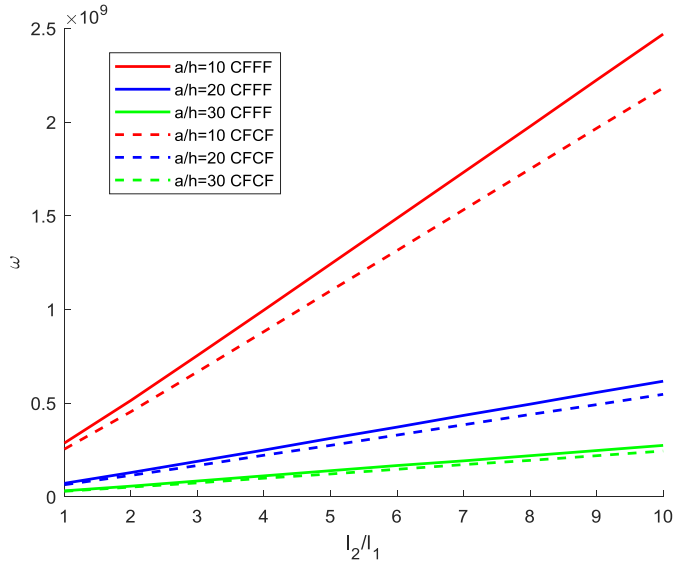

Fig. (7) Natural frequencies of CFFF and CFCF microplate with a different MLSPR

5. Results verification

In this section, the results of this study are compared with the Ritz method [23]. For simplification, the effects of the second layer are ignored and the system is modeled as a one-layer CFCF microplate. Also, the material length scale parameter is assumed 0. Figure 8 illustrates the natural frequency according to the width-to-thickness ratio. The difference between results based on the

two methods is large when the width of the plate is small. However, the discrepancy is decreased as the width increases. This method seems to give better answers for larger plates.

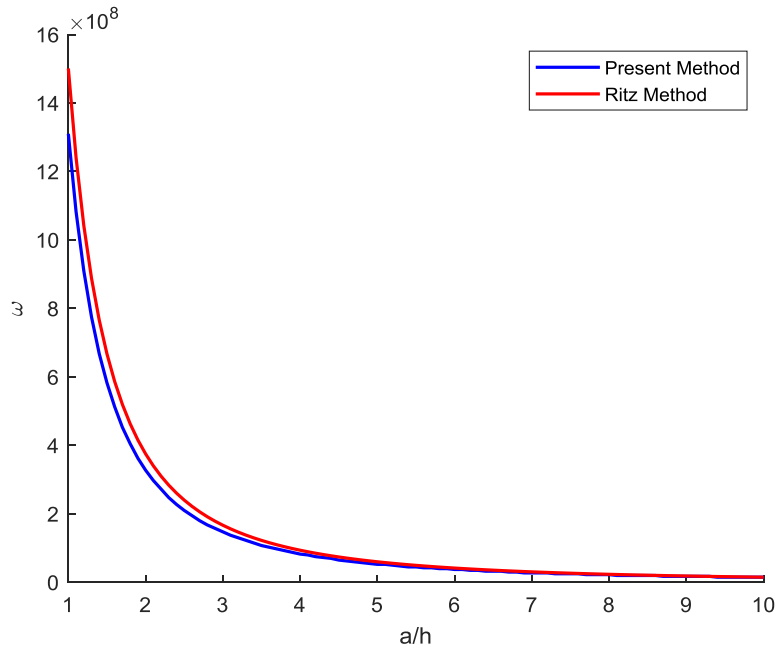

Fig. (8) Comparing the results of the present method with the results of the Ritz method for a CFCF plate

6. Conclusion

In this paper, the dynamic behavior of a microsensor, modeled as a two-layer microplate, was investigated with an analytical approach. Modified Couple Stress Theory (MCST) is used to record the effect of material size on the extraction of governing equations and natural frequency in this paper. Also, the first natural frequency of the system is extracted using the Rayleigh-Ritz method. The Rayleigh-Ritz method uses a penalty approach for the first time and the fixed boundaries of microplate or simply supported boundaries are modeled as springs. The reason for using the penalty approach is the use of this method in different boundary conditions. The natural frequency of the system is extracted at different thicknesses of the silicon layer as well as for different materials of the second layer.

The first natural frequency of the system is extracted according to different system parameters. The results show that the natural frequency decreases when the thickness of the second layer increases for different (a/h) ratios. Also, despite the different first natural frequencies in different parameters, the natural frequency diagram shows the same behavior in terms of system parameters in different boundary conditions.

Based on the penalty approach, the increase of the material length scale parameter can increase the natural frequency and the boundary conditions do not have a significant effect on this increase. This increase is larger when the width of the square plate is small. Also, it can be inferred from the results and verification that by using this method the results agree with the results of the previous studies as the width of the microplate increases.

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